Intermediate Algebra Summary - Part I

This is an overview of the key ideas we have discussed during the first part of this course. You may find this summary useful as a study aid, but remember that the only way to really master the skills and understand the ideas is to practice solving problems.

The Addition Method (For Solving Systems of Linear Equations)
Write both equations in the form $ax + by = c$; multiply each equation by a constant, if necessary, so that the coefficients of one of the variables are the same size; then either add or subtract the two equations in order to eliminate one of the variables; solve for the remaining variable; then plug that value back into one of the original equations to solve for the other unknown.

**Example:** Solve the system of linear equations:

$$2x + 5y = 8$$
$$3x + 4y = 1$$

**Solution:** Begin by multiply the first equation by 3 and the second equation by 2, so that the coefficients of $x$ match up:

$$6x + 15y = 24$$
$$6x + 8y = 2$$

Then subtract the second equation from the first to eliminate $x$:

$$7y = 22.$$ 

Hence

$$y = \frac{22}{7}.$$ 

Now we plug this into the original first equation to get

$$2x + 5 \left( \frac{22}{7} \right) = 8$$

Therefore

$$2x + \frac{110}{7} = 8.$$
Then we isolate $x$:

$$2x = 8 - \frac{110}{7}$$

so

$$2x = \frac{56}{7} - \frac{110}{7}$$

therefore

$$2x = -\frac{54}{7}$$

and consequently

$$x = -\frac{54}{7} \cdot \frac{1}{2} = -\frac{27}{7}.$$  

So the answers are 

$$x = -\frac{27}{7} \quad \text{or} \quad y = \frac{22}{7}$$

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**Solving Absolute Value Equations**

**Example:** Solve the equation $|3x + 4| = 2$.

**Solution:** Either

$$3x + 4 = 2 \quad \text{or} \quad 3x + 4 = -2$$

Hence

$$3x = -2 \quad \text{or} \quad 3x = -6$$

Therefore

$$x = -\frac{2}{3} \quad \text{or} \quad x = -2$$

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**The Point-Slope Formula for a Line**

**Example:** Find the equation for a line through the points $(2, 5)$ and $(-3, 2)$.

**Solution:** First we find the slope:

$$m = \frac{2 - 5}{-3 - 2} = \frac{-7}{-5} = \frac{7}{5}$$

Now we plug this slope and either point into the formula $y = y_1 = m(x - x_1)$ to get

$$y - 5 = \frac{7}{5}(x - 2)$$

That is the equation for the line. If we need to, we can simplify and solve for $y$:

$$y - 5 = \frac{7}{5}x - \frac{14}{5}$$
so

\[ y = \frac{7}{5}x + \frac{11}{5} \]

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**Solving Quadratic Equations by Factoring**

**Example:** Solve the equation \( x^2 = 5x + 24 \) by factoring.

**Solution:** First, we need to move all the terms to one side of the equation, so we subtract 5x from both sides, and we subtract 24 from both sides to get

\[ x^2 - 5x - 24 = 0 \]

Now we factor the left side:

\( (x - 8)(x + 3) = 0 \)

The only way this can be true is if one of the two factors is zero, so we see that either

\[ x - 8 = 0 \quad \text{or} \quad x + 3 = 0 \]

Consequently,

\[ x = 9 \quad \text{or} \quad x = -3. \]

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**Solving Quadratic Equations by Completing the Square**

**Example:** Solve the equation \( 3x^2 = 10x + 6 \) by completing the square.

**Solution:** We need to get all the terms with \( x \) or \( x^2 \) on the same side, so we subtract 10x from both sides:

\[ 3x^2 - 10x = 6 \]

We also need the coefficient of \( x \) to be 1, so we divide both sides of the equation by 3:

\[ x^2 - \frac{10}{3}x = 2 \]

Now we complete the square: the coefficient of \( x \) on the left side is \(-\frac{10}{3}\); half of this is \(-\frac{5}{3}\); the square of \(-\frac{5}{3}\) is \(\frac{25}{9}\), so that’s what we need to add to both sides:

\[ x^2 - \frac{10}{3}x + \frac{25}{9} = 2 + \frac{25}{9} \]

Since \( 2 = \frac{18}{9} \), we can write this as

\[ x^2 - \frac{10}{3}x + \frac{25}{9} = \frac{43}{9} \]

Now we factor the left side:

\[ (x - \frac{5}{3})^2 = \frac{43}{9} \]
Then take square roots of both sides:

$$\sqrt{\left( x - \frac{5}{3} \right)} = \sqrt{\frac{43}{9}}$$

Therefore

$$\left| x - \frac{5}{3} \right| = \sqrt{\frac{43}{9}}$$

Hence either

$$x - \frac{5}{3} = \sqrt{\frac{43}{9}} \quad \text{or} \quad x - \frac{5}{3} = -\sqrt{\frac{43}{9}}$$

so

$$x = \frac{5}{3} + \sqrt{\frac{43}{9}} \quad \text{or} \quad x = \frac{5}{3} - \sqrt{\frac{43}{9}}$$

Function Notation

Example: Suppose that a function is given by the formula \( f(x) = x^2 - 2x \). Find \( f(2) \) and \( f(-3) \).

Solution: We evaluate the function by plugging the appropriate value in for \( x \) and simplifying:

\[
\begin{align*}
  f(2) &= (2)^2 - 2(2) \\
  &= 4 - 4 \\
  &= 0
\end{align*}
\]

Similarly,

\[
\begin{align*}
  f(-3) &= (-3)^2 - 2(-3) \\
  &= 9 - 6 \\
  &= 3
\end{align*}
\]

So we have

$$f(2) = 0 \quad \text{and} \quad f(-3) = 3$$

Example: Find a formula for a linear function \( f(x) \) that satisfies \( f(2) = 7 \) and \( f(4) = -1 \).

Solution: First we find an equation for a line that passes through the points \((2, 7)\) and \((4, -1)\) – we can do this using the point-slope formula:

\[
\begin{align*}
  m &= \frac{-1 - 7}{4 - 2} \\
  &= \frac{-8}{2} \\
  &= -4
\end{align*}
\]
Therefore \[ y - (-1) = -4(x - 4) \]

so \[ y + 1 = -4x + 16 \]

Therefore \[ y = -4x + 15 \]

Now we replace the \( y \) with \( f(x) \) to obtain function notation:

\[ f(x) = -4x + 15 \]
ENTERING LISTS

Most statistical calculations involve working with tables of data, so we need to be able to input such tables into the graphing calculator. In the following example, we will enter the data in the table at right. The same data will be used in the later examples in this guide.

1. Select the STAT menu by pressing:

2. Select the first item in order to edit the calculator’s lists:

3. Enter the x-values in the first column, labeled L1, as follows:

4. Use the right arrow key to move the cursor to the first row of the second column (labeled L2); then enter the y-values as follows:
SCATTERPLOTS

You can plot individual points on an xy-plane by performing a scatterplot.

1. First, set the window as you would when graphing a function. To get to the menu for this, press:

   ![WINDOW]

   

   WINDOW
   Xmin=0
   Xmax=15
   Xscl=1
   Ymin=0
   Ymax=15
   Yscl=1
   Xres=1

2. Go to the STAT PLOT menu by pressing:

   ![2nd Y=]

   2nd Y=

   STAT PLOT
   1:Plot1...Off
   L< L1 L2
   2:Plot2...Off
   L< L1 L2
   3:Plot3...Off
   L< L1 L2
   4:PlotsOff

3. Select the first plot by pressing 1 or enter:

   ![ENTER]

   ENTER

   PLOT
   Plot1 Plot2 Plot3
   On On Off
   Type: sc L1 L2
   Xlist:L1
   Ylist:L2
   Mark: F + .

4. Select On by pressing enter:

   ![GRAPH]

   GRAPH

   PLOT
   Plot1 Plot2 Plot3
   Off Off Off
   Type: sc L1 L2
   Xlist:L1
   Ylist:L2
   Mark: F + .

5. View your plot by pressing:

   ![GRAPH]
REgressions

You can fit a line or curve to a set of data points by performing a regression with the graphing calculator. In this example, we find a line of best fit using a linear regression.

1. Go to the STAT menu by pressing:

   

2. Use the right arrow key to select the CALC menu:

   

3. Select LinReg(ax+b) to perform a linear regression on the data in lists L1 (for x-values) and L2 (for y-values):

   

4. Calculate the coefficient for the regression by pressing:

   

5. The resulting screen gives you the coefficients for the regression, as well as the correlation coefficient r and its square.

   

   ```plaintext
   LinReg
   y=ax+b
   a=.3279808678
   b=5.812435941
   r²=.4664616786
   r=.6829799987
   ```