Sample Questions for Final Exam

1. Simplify the expression $\frac{4}{x+2} - \frac{2}{x+1}$. (Section 8.3)

\[
\frac{4}{(x+2)(x+1)} - \frac{2(x+2)}{(x+1)(x+2)} = \frac{4x+4 - 2x - 4}{(x+1)(x+2)} = \frac{2x}{(x+1)(x+2)}
\]

2. Simplify the expression $\frac{1}{1-x} + \frac{x^2}{x-1}$. (Section 8.3)

\[
\frac{1}{1-x} + \frac{x^2}{x-1} = \frac{-1}{x-1} + \frac{x^2}{x-1} = \frac{x^2-1}{x-1} = \frac{(x+1)(x-1)}{x-1} = x+1
\]

3. Solve the equation $\frac{1}{x+1} + \frac{1}{x} = 1$. (Section 8.5)

\[
x(x+1) \left( \frac{1}{x+1} + \frac{1}{x} \right) = x(x+1) \Rightarrow 0 = x^2 - x - 1
\]

\[
\Rightarrow x = \frac{1 \pm \sqrt{(-1)^2 - 4(-1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}
\]

4. Simplify the expression $(x^{-1}y^3)(x^3y^{-2})$. (Section 9.5)

\[
x^{-1}x^3 \cdot y^3 \cdot y^{-2} = x^{(-1+3)} y^{3-2} = x^2 y
\]
5. Simplify the expression $\frac{x^2}{\sqrt{x}}$. (Section 9.3)

\[ \frac{x^2}{\sqrt{x}} = \frac{x^2}{x^{\frac{1}{2}}} = x^{2-\frac{1}{2}} = x^{\frac{3}{2}} \]

6. Simplify the expression $\left(\frac{\sqrt{x^2-x}}{x^2-18x}\right)^0$. (Section 9.5)

\[ \frac{\sqrt{x^2-x}}{x^2-18x} \]

7. Simplify the expression $\sqrt{48} - \sqrt{12}$. (Section 9.2)

\[ \sqrt{2\cdot2\cdot2\cdot3} - \sqrt{2\cdot2\cdot3} = 2\cdot2\sqrt{3} - 2\sqrt{3} = 4\sqrt{3} - 2\sqrt{3} \]
\[ = 2\sqrt{3} \]

8. Show that $x = 1 - \sqrt{3}$ satisfies $x^2 - 2x - 2 = 0$. (Sections 9.2-9.3)

\[ (1-\sqrt{3})^2 - 2(1-\sqrt{3}) - 2 = 1 - 2\sqrt{3} + 3 - 2 + 2\sqrt{3} - 2 \]
\[ = 1 - 2 - 2 \]
\[ = 0 \]

9. Solve the equation $\sqrt{5x - 1} = 2$, and check your answer. (Section 9.4)

\[ 5x - 1 = 2^2 \]
\[ 5x - 1 = 4 \]
\[ 5x = 5 \]
\[ x = 1 \].
10. Use the quadratic formula to find the solutions of the equation $2x^2 + 2x = -1$ and simplify the answers. (Section 7.4)

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)} = \frac{-2 \pm \sqrt{4}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

11. Find the coordinates of the vertex of the parabola $y = x^2 - 2x - 1$. (Section 7.5)

$$x = \frac{-(-2)}{2(1)} = 1$$

$$y = 1^2 - 2(1) - 1 = -2$$

$$\boxed{(1, -2)}$$

12. When a small company sells $x$ MP3 players, it’s total profit in dollars is $y = 500x - 5x^2$. How many MP3 players should the company sell to get the maximum profit? (Section 7.5)

$$x = \frac{-500}{2(-5)} = \frac{500}{10} = 50$$

13. Sixty miles per hour is how many feet per second? (Hint: There are 5280 feet in each mile.) (Supplemental Handout)

$$\frac{60 \text{ miles}}{1 \text{ hr}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 88 \frac{\text{ ft}}{\text{ sec}}$$

14. Simplify the expression $\frac{ab + b}{a^2 + a}$, and state any restrictions on the variables $a$ and $b$. (Section 8.2)

$$\frac{ab + b}{a^2 + a} = \frac{b(a + 1)}{a(a + 1)} = \frac{b}{a}$$
15. Is the function \( f(x) = 3x(1 - x) \) linear, quadratic, or neither? (Section 7.3)

\[
\begin{align*}
    f(x) &= 3x - 3x^2 \\
    \text{This is quadratic.}
\end{align*}
\]

16. What are the \( x \)-intercepts of the equation \( y = (2x + 1)(4 - x) \)? (Section 7.4)

\[
\begin{align*}
    0 &= (2x+1)\,(4-x) \\
    \Rightarrow \quad 2x+1 &= 0 \quad \text{or} \quad 4-x = 0 \\
    \Rightarrow \quad 2x &= -1 \quad \text{or} \quad 4 = x \\
    \Rightarrow \quad x &= -\frac{1}{2} \quad \text{or} \quad x = 4
\end{align*}
\]

17. Find the equation of a line that goes through the points \((-1, 2)\) and \((2, 0)\). (Section 7.3)

Use the point-slope formula:

\[
y - y_1 = m(x - x_1)
\]

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2}{2 - (-1)} = -\frac{2}{3}
\]

\[
\Rightarrow \quad y - 0 = -\frac{2}{3}(x - 2) \quad \Rightarrow \quad y = -\frac{2}{3}x + \frac{4}{3}
\]

18. Find the equation of a line through the point \((1, 2)\) that is parallel to the line \(x + y = 1\). (Section 7.3)

This line is parallel, so we need \(m = -1\), \((x, y) = (1, 2)\).

\[
\Rightarrow \quad y - 2 = -1(x - 1) \quad \Rightarrow \quad y = -x + 3
\]
19 A popular company rents moving trucks for $29 plus 99 cents per mile driven. Write down a function that describes the cost in dollars of renting a truck and driving it for \( x \) miles. \((\text{Section 7.3})\)

\[ y = 29 + 0.99x \]

20 If \( y \) varies inversely with \( x \) and \( y = 2 \) when \( x = 5 \), find an equation for \( y \) in terms of \( x \). \((\text{Section 8.6})\)

\[ y = \frac{k}{x} \]

\[ 2 = \frac{k}{5} \]

\[ 10 = k \]

\[ \Rightarrow \quad y = \frac{10}{x} \]

21 Simplify the expression \( \frac{x^2 + 2x + 1}{x^2 - 1} \div \frac{x + 2}{x - 1} \) completely. \((\text{Section 8.2})\)

\[ \frac{(x+1)(x+1)}{(x+1)(x-1)} \div \frac{x+2}{x-1} = \frac{(x+1)(x+1)}{(x+1)(x-1)} \cdot \frac{x-1}{x+2} \]

\[ = \frac{x+1}{x+2} \]

22 Solve the equation \( \sqrt{3x - 5} = 4 \), and find any restrictions on the variable. \((\text{Section 9.4})\)

Restrictions: The expression under the square root can't be negative, so

\[ 3x - 5 \geq 0 \quad \Rightarrow \quad 3x \geq 5 \quad \Rightarrow \quad x \geq \frac{5}{3} \]

Solution

\[ \sqrt{3x - 5} = 4 \quad \Rightarrow \quad 3x - 5 = 16 \]

\[ \Rightarrow \quad 3x = 21 \]

\[ \Rightarrow \quad x = 7 \]