ALE 2. Significant Figures

CHEM 161 – K. Marr

Name ______________________________
Team No. ________ **Section** __________

What are significant figures, what do they indicate and how are they used in addition, subtraction, multiplication and division?

The Model

(Reference: Section 1.6 in *Silberberg 5th ed.*)

There are two kinds of numbers in the world**:**

Exact numbers

- There are exactly 12 eggs in a dozen.
- Most people have exactly 10 fingers and 10 toes.
- 1 meter = 100 centimeters.
- 1 yard = 36 inches
- 1 dollar = 100 cents = 4 quarters
- 1 kilometer = 1000 meters

Inexact numbers:

- ***Any measured value***: use of a 10 mL graduated cylinder to measure the volume of a solution might give a volume of 8.81 mL (3 significant figures) or a less precise volume of 8.7 mL with a 100 mL graduated cylinder.
- An analytical balance might find the mass of a pencil to be 12.1403 g (6 sig. figs.), while a centigram balance might find it to weigh 12.13 g (4 sig. figs.)

The number of digits, i.e. significant figures, reported for a ***measured value*** conveys the quality of the measurement and hence, the quality of the measuring device. It is important to use significant figures correctly when reporting a measurement so that it does not appear to be more (or less) precise than the equipment used to make the measurement allows. We can achieve this by controlling the number of digits, or **significant figures**, used to report the measurement.

In this course and in others, *you **must** use correct significant figures in reporting your results*. Laboratory measuring instruments have their limits, just as your senses have their limits. One of your tasks, in addition to learning how to use various measuring instruments properly, will be to correctly determine the precision of the measuring devices that you use and to *report all measured and calculated values to the correct number of significant figures*.

Significant Figure Rules

There are three rules on determining how many significant figures are in a number:

1. **Non-zero digits are always significant.** (see page 2 for details)
2. **Any zeros between two significant digits are significant.** (see page 2 for details)
3. **A final zero or trailing zeros in the decimal portion ONLY are significant.** (see pages 2-3)

Focus on these rules and learn them well. They will be used extensively throughout Chem 161, Chem 162 & Chem 163. You would be well advised to do as many problems as needed to nail the concept of significant figures down tight and then do some more, just to be sure.

Please remember that, in science, with the exception of a few numbers that are defined and hence exact, all numbers are based upon measurements. Since all measurements are uncertain, we must only use those numbers that are meaningful. A common ruler cannot measure something to be 22.4072643 cm long. Not all of the digits have meaning (significance) and, therefore, should not be written down. In science, only the numbers that have significance (derived from measurement) are written.

Rule 1: Non-zero digits are always significant.

Hopefully, this rule seems rather obvious. If you measure something and the device you use (ruler, thermometer, triple-beam balance, etc.) returns a number to you, then you have made a measurement decision and that ACT of measuring gives significance to that particular numeral (or digit) in the overall value you obtain.

Hence a number like 26.38 would have four significant figures and 7.94 would have three. The problem comes with numbers like 0.00980 or 28.09.

Rule 2: Any zeros between two significant digits are significant. (i.e. “sandwiched” zeroes are significant)

Suppose you have a measured value like 406. By the first rule, above, the 4 and the 6 are significant. However, to make a measurement decision on the 4 (in the hundred's place) and the 6 (in the one’s place), you HAD to have made a decision on the ten's place. The measurement scale for this number would have calibration marks for the hundreds and tens places with an estimation made in the “ones” place—hence, ***significant figures indicate the number of digits known with certainty*** (e.g. the 1st two digits in 406) ***and one that is an estimate*** (e.g. the 6 in 406). Such a measuring measurement scale would look like this:

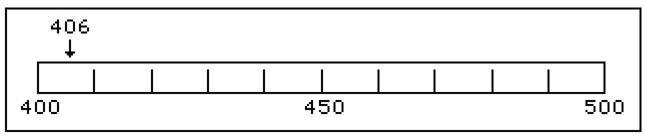

Figure 1. A measuring scale that allows one to use three significant figures

Rule 3: A final zero or trailing zeros in the decimal portion ONLY *are* significant.

This rule causes the most difficulty with students. Here are two examples of this rule with the zeros this rule affects in bold font:

0.005**00**

0.0304**0**

Here are two more examples where the significant zeros are in bold font:

$2.3\mathbf{0} \times 10^{-5}$

$4.5\mathbf{00} \times 10^{12}$

Zeros not Discussed Above

Zero Type #1. Space holding zeros on numbers less than one.

Here are the first two numbers used under rule 3, above. The digits that are NOT significant are underlined:

$$0.\underline{00}500 = 5.00 \times 10^{-3}$$

$$0.\underline{0}3040 = 3.040 \times 10^{-2}$$

The underlined zeroes serve only as space holders—their function is to locate the decimal point. They DO NOT involve measurement decisions. The non-significant zeros disappear upon writing the numbers in scientific notation.

Zero Type #2. The zero to the left of the decimal point on numbers less than one.

When a number like 0.00500 is written, the very first zero (to the left of the decimal point) is put there by convention. Its sole function is to communicate unambiguously that the decimal point is a decimal point. If the number were written like this, .00500, there is a possibility that the decimal point might be mistaken for a period. Many students omit that zero. They should not.

Zero Type #3. Trailing zeros in a whole number *without* a decimal point are *not* significant.

200 has only *one* significant figure 25,000 has *two* sig figs

This is based on the way each number is written. When whole number are written as above, the zeros, BY DEFINITION, did not require a measurement decision, thus they are not significant. However, it is entirely possible that 200 really does have two or three or more significant figures. If it does, it will be written differently. Typically, scientific notation, underlining or the use of a decimal point is used for this purpose.

2 significant figures: $2\underline{0}0$ or 2.0×10^2

3 significant figures: 200. or $20\underline{0}$ or 2.00×10^2

4 significant figures: 200.0 or 2.000×10^2

How will you know how many significant figures are in a number like 200? In a problem without a scientific context, you should be told. If you were doing an experiment, the context of the experiment and its measuring device would tell you how many significant figures to report to people who read the report of your work.

Exact Numbers

Exact numbers, such as the number of people in a room, have an infinite number of significant figures. Exact numbers are counting up how many of something are present, they are not measurements made with instruments. Another example of this are defined numbers, such as 1 foot = 12 inches. There are exactly 12 inches in one foot. Therefore, if a number is exact, it DOES NOT affect the precision of a calculation.

Some more examples:

There are 100 years in a century.

2 molecules of hydrogen react with 1 molecule of oxygen to form 2 molecules of water.

1 kilogram = 1000 grams

Each molecule of methane gas, CH_4, contains exactly 1 carbon atom and 4 hydrogen atoms.

Interestingly, the speed of light is now a defined quantity. By definition, the value is 299,792,458 meters per second.

A Brief Aside

There might come an occasion in chemistry when you are not exactly sure how many significant figures are called for. Suppose the textbook mentions 100 mL. You look at this and see only one significant figure. However, an experienced chemist would know that 100 mL can be easily measured to 3 or 4 significant figures. Why then doesn't the textbook (or the professor) write 100.0 (for 4 sig figs) or 1.00 x 10^2 (for 3 sig figs)? The textbook writer or the professor might be assuming that all in his or her audience understands these matters and so it is no big deal to simply write 100. Or…. they are lazy.

So, a <u>*brief word of advice*</u>: If you haven't a clue as to how many significant figures to use, try using three or four. These are reasonable numbers of significant figures for most chemical activities. Also, look out for the instructor who ignores significant figures, then makes a big deal of it on a test. Forewarned is forearmed!

Key Questions

1. What kind of numbers are exact numbers? Give at least one *original* example.

2. What kind of numbers are inexact numbers? Why? Give at least one *original* example.

3. What is your understanding of significant figures: What are significant figures, when should they be used and what function do they serve?

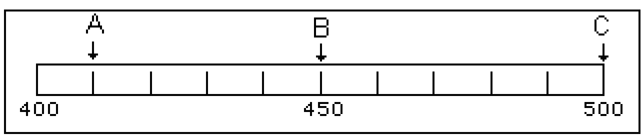

Figure 2. A hypothetical measuring scale

4. What values would you record for measurements A, B and C if each measurement fell on the line each arrow points to in figure 2, above? How many sig figs should each measurement have?

   A =________________ B =________________ C = ________________

5. Later in the quarter you will be asked to measure out accurately about 3 grams of an unknown salt with a milligram electronic balance (a balance that measures out to the nearest milligram, 0.001 g). What mass of salt should you measure out? How many significant figures should you record?

6. Suppose you are asked to measure out about 25 mL of deionized water <u>*as accurately as you can*</u>.
   a.) What measuring device would you use?________________________________

   b.) How much water should you measure out? _____________________________

   c.) How many significant figures would you report? ______________________

Exercises

7. How many significant figures are there in each of the following numbers? Record your responses in the spaces provided and circle the digits that are significant.

a.) 3.0800	____________	f.) 3.200×10^{9}	____________
b.) 0.00418	____________	g.) 250	____________
c.) 7.09×10^{-5}	____________	h.) 780,000,000	____________
d.) 91,600	____________	i.) 0.0101	____________
e.) 0.003005	____________	j.) 0.00800	____________

Model: Rounding Numbers

In numerical problems, it is often necessary to round numbers to the appropriate number of significant figures. Consider the following examples in which each number is rounded so that each of them contains 4 significant figures. Study each example and make sure you understand why they were rounded as they were:

Original number	**→**	**Number rounded to 4 sig figs**
41,008	→	41,010
1.25624	→	1.256
0.017837	→	0.01784
120	→	120.0
127.450	→	127.4
127.4501	→	127.5
127.550	→	127.6
127.25000	→	127.2
127.25001	→	127.3
127.35	→	127.4

Key Questions

8. Summarize the rounding rule(s) used in the *first three examples*, above.

9. Summarize the rounding rule(s) used in the last six examples (i.e. those using 127), above. This rule is often referred to as the "odd – even" rule.

Exercises

10. Round the following numbers to four significant figures.

a.) 2.16347×10^5 =

b.) 4.000574×10^6 =

c.) 3.6825 =

d.) 7.2518 =

e.) 375.6523 =

f.) 21.865001 =

11. Round off each number to the indicated number of significant figures (sf).

a.) 231.554 (to 2 sf) =

b.) 0.00845 (to 2 sf) =

c.) 150,000 (to 1 sf) =

d.) 0.0023 (to 3 sf) =

e.) 249,441 (to 3 sf) =

f.) 0.00250122 (to 3 sf) =

g.) 12,049,002 (to 4 sf) =

h.) 0.00200210 (to 3 sf) =

The Model: Using Significant Figures in Addition and Subtraction

Did you know that 30,000 plus 1 does not always equal 30,001? In fact, 30,000 + 1 is sometimes equal to 30,000! You may find this hard to believe, but let's examine this.

Recall that zeros in a number are not always *significant*. Knowing this makes a big difference in how we add and subtract. For example, consider a swimming pool that can hold 30,000 gallons of water. If I fill the pool to the maximum fill line and then go and fill an empty one gallon milk jug with water and add it to the pool, do I then have exactly 30,001 gallons of water in the pool? Of course not. I had approximately 30,000 gallons before and after I added the additional gallon because "30,000 gallons" is not a very precise measurement. So we see that sometimes 30,000 + 1 = 30,000!

In mathematical operations involving significant figures, the answer is reported in a way that reflects the reliability of the <u>least precise</u> number. ***An answer is no more precise that the least precise number used to get the answer***. Imagine a team race where you and your teammates must finish together at the same time. Who dictates the speed of the team? Of course, the slowest member of the team. Your answer cannot be MORE precise than the least precise measurement.

Use the "decimal rule" when adding and subtracting numbers:

> For addition or subtraction, the answer must be rounded off to contain only as many decimal places as are in the value with the *<u>least</u>* number of decimal places.

WARNING!! The rules for addition/subtraction are different from those of multiplication/division. A very common student error is to swap the two sets of rules. Another common error is to use just one rule for both types of operations.

Example #1. **350.04 + 720 = 1070.04 = 1070**

$$\begin{array}{r} 350.04 \\ +720 \\ \hline 1070.04 \\ 1070 \end{array}$$

This number is precise to the hundredths place.

This number is precise to the tens place.

The answer can only be as precise as the *least* precise number in the operation. Hence the answer is rounded off to the tens place since the tens place is less precise than the hundredths place.

Example #2. **7000 - 1770 = 5230 = 5000**

$$\begin{array}{r} 7000 \\ -\ 1770 \\ \hline 5230 \\ 5000 \end{array}$$

This number is precise to the tens place

This number is precise to the thousands place.

The answer can only be as precise as the *least* precise number in the operation. Hence the answer is rounded off to the thousands place since the thousands place is less precise than the tens place.

Key Questions

12. Consider example #1 from above. Indicate in the spaces below the number of significant figures (sf) for each number in the problem.

350.04	+	720	=	1070.04	=	1070
____sf		**____sf**		**____sf**		**____sf**

Should the number of significant figures be considered when adding or subtracting measured numbers? *Explain*

13. When you add and subtract numbers, how do you identify the first uncertain number in the result?

Exercises

Record the answer before and after rounding off for each problem below.

14.) 3.461728 + 14.91 + 0.980001 + 5.2631 = ________________ = ________________

15.) 23.1 + 4.77 + 125.39 + 3.581 = __________________ = _______________________

16.) 22.101 - 0.9307 = ____________________ = _______________________

17.) 0.04216 - 0.0004134 = ______________________ = _______________________

18.) 564,321 - 264,321 =________________________ = _________________

The Model: Using Significant Figures in Multiplication and Division

A chain is no stronger than its weakest link—that is, an answer is no more precise that the least precise number used to get the answer.

Use the "Chain Rule" when multiplying and dividing measured numbers:

> When measurements are multiplied or divided, the answer can contain no more significant figures than the number with the fewest number of significant figures. This means you MUST know how to recognize significant figures in order to use this rule.

To round correctly, follow these simple steps:

1) Count the number of significant figures in each number.
2) Round your answer to the least number of significant figures.

Example #1.

3 sig figs

$$\frac{4560}{14} = 325.714285714 = 330$$

2 sig figs

The rounded answer has only 2 significant figures since 2 is the least number of significant figures in this problem.

Example #2.

$$13.1 \times 1.2039 = 15.77109 = 15.8$$

3 sig figs 5 sig figs

The rounded answer has 3 significant figures since 3 is the least number of significant figures in this problem.

Multi-Step Calculations: Keep at Least One Extra Significant Figure in Intermediate Answers

When doing multi-step calculations, *keep at least one more significant figure in intermediate results* than needed in your final answer. For example, if a final answer requires two significant figures, then carry *at least* three significant figures in all calculations. If you round-off all your intermediate answers to only two digits, you are discarding the information contained in the third digit, and as a result the *second* digit in your final answer might be incorrect. This phenomenon is known as "round-off error." <u>*Avoid rounding errors by carrying at least on extra sig fig throughout a multi-step calculation and then round off to the correct number of sig figs at the very end.*</u>

Key Questions

19. When you multiply and divide numbers, what is the relationship between the number of significant figures in the result and the number of significant figures in the numbers you are multiplying or dividing?

Exercises

Record the answer *before* and *after* rounding off for each problem below.

20.) $(3.4617 \times 10^{7}) \div (5.61 \times 10^{-4})$ = ______________ = ________________

21.) $[(9.714 \times 10^{5})(2.1482 \times 10^{-9})] \div [(4.1212)(3.7792 \times 10^{-5})]$ = ________________________

(Watch your order of operations on this problem!) = ________________________

22.) $(4.7620 \times 10^{-15}) \div [(3.8529 \times 10^{12})(2.813 \times 10^{-7})(9.50)]$ = ________________________

= ________________________

23.) $[(561.0)(34{,}908)(23.0)] \div [(21.888)(75.2)(120.00)]$ = ______________________________

= ______________________________

24. Carry out each of the following calculations. *Check that each answer has the correct number of significant figures and the correct units of measure*.

a.) $\dfrac{2.420\,g + 15.6\,g}{4.8\,g} =$

b.) $\dfrac{7.87\,g}{16.1\,mL - 8.44\,mL} =$

c.) $V = \pi r^2 h \quad where\ r = 6.23\ cm$ and $h = 4.630\ cm$

$V =$

d.) $\dfrac{8.32 \times 10^{7} g}{\frac{4}{3}(3.1416)(1.95 \times 10^{2} cm)^{3}} =$

Note: 4/3 is an exact number!

e.) $E_k = \frac{1}{2}mv^2 = \dfrac{(1.84 \times 10^{2} g)(44.7\ ^{m}/_{s})^{2}}{2} =$

f.) $\dfrac{(1.07 \times 10^{-4} \frac{mol}{L})^{2}\,(2.6 \times 10^{-3} \frac{mol}{L})}{(8.35 \times 10^{-5} \frac{mol}{L})(1.48 \times 10^{-2} \frac{mol}{L})^{3}} =$