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# ALE 7. Equilibrium Constants 

(Reference: 17.3 Silberberg $5^{\text {th }}$ edition)

## Why does a chemical reaction have more than one equilibrium constant?

## The Model: Relationship Between $\boldsymbol{K}_{\mathrm{c}}$ and $\boldsymbol{K}_{\mathrm{p}}$

Many of the systems in which there is a reversible reaction involve gases. Assuming that a gas behaves ideally (i.e., ${ }^{n} /{ }_{V}={ }^{P} / R T$ ) and that the temperature is constant, then the molar concentration of a gas is directly proportional to the partial pressure of the gas in the reaction mixture. Therefore it is often convenient to monitor the pressures of gases rather than their molar concentrations. The problem with this is that it requires more than one equilibrium constant to describe where the masses (of reactants and products) will be when the reversible reaction reaches equilibrium. If we are interested in the equilibrium molar concentrations, then we need $\boldsymbol{K}_{\boldsymbol{c}}$, in which the law of mass action involves a ratio of concentrations. And if we are interested in the equilibrium partial pressures, then we need $\boldsymbol{K}_{\boldsymbol{p}}$, in which the law of mass action involves a ratio of pressures. The relationship between $K_{\mathrm{c}}$ and $K_{\mathrm{p}}$ is shown in equation $\mathbf{1}$.

$$
\begin{equation*}
K_{c}=K_{p}(R T)^{-\Delta n} \tag{1}
\end{equation*}
$$

Here $\Delta \boldsymbol{n}$ is the difference between the stoichiometric coefficients of the gaseous products and reactants, $\boldsymbol{T}$ is the absolute temperature of the reaction mixture, and $\boldsymbol{R}=0.08206 \mathrm{~K}^{-1}$. (We assume that partial pressures of the gases are in atm and the units of the gas constant are simplified so that equilibrium constants remain unitless.)

As an example, consider the decomposition reaction of dinitrogen tetroxide to become nitrogen dioxide.

$$
\begin{equation*}
\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g}) \rightleftharpoons 2 \mathrm{NO}_{2}(g) \tag{2}
\end{equation*}
$$

The laws of mass action in terms of molar concentrations (eqn $\mathbf{2 a}$ ) and in terms of partial pressures (eqn $\mathbf{2 b}$ ) are as follows.

$$
\begin{equation*}
K_{c}=\frac{\left[\mathrm{NO}_{2}\right]^{2}}{\left[\mathrm{~N}_{2} \mathrm{O}_{4}\right]} \tag{2a}
\end{equation*}
$$

$$
\begin{equation*}
K_{p}=\frac{P_{\mathrm{NO}_{2}}^{2}}{P_{\mathrm{N}_{2} \mathrm{O}_{4}}} \tag{2b}
\end{equation*}
$$

Since $\left[\mathrm{NO}_{2}\right]=\frac{P_{\mathrm{NO}_{2}}}{R T}$ and $\left[\mathrm{N}_{2} \mathrm{O}_{4}\right]=\frac{P_{\mathrm{N}_{2} \mathrm{O}_{4}}}{R T}$, eqn 2a can be written as eqn 2c.

$$
\begin{equation*}
K_{c}=\frac{\left(\frac{P_{\mathrm{NO}_{2}}}{R T}\right)^{2}}{\frac{P_{\mathrm{N}_{2} \mathrm{O}_{4}}}{R T}} \tag{2c}
\end{equation*}
$$

Algebraically simplifying eqn $\mathbf{2 c}$ and using eqn $\mathbf{2 b}$ as a substitution in eqn $\mathbf{2 c}$ leads to eqn $\mathbf{2 d}$. For

$$
\begin{equation*}
K_{c}=K_{p}(R T)^{-1} \tag{2d}
\end{equation*}
$$

## Key Questions

1 a. What is the difference between the stoichiometric coefficients of the products and reactants in the decomposition of dinitrogen tetroxide (eqn 2 in the Model)? Show your work.
$\Delta n=$
b. What is the power to which $R T$ is raised in eqn $\mathbf{2 d}$ ? Does eqn $\mathbf{2 d}$ comply with eqn $\mathbf{1}$ ?
2. When would $K_{\mathrm{c}}$ be equal to $K_{\mathrm{p}}$ for the same reaction? Explain your answer.

## Exercises

3. Calculate $K_{\mathrm{c}}$ for the decomposition of phosgene at $395^{\circ} \mathrm{C}$.

$$
\mathrm{COCl}_{2}(g) \rightleftharpoons \mathrm{CO}(g)+\mathrm{Cl}_{2}(g) \quad K_{\mathrm{p}}=0.0451 \text { at } 395^{\circ} \mathrm{C} .
$$

4. What is the value of $K_{\mathrm{p}}$ for formation of hydrogen iodide from its elements at $425.4^{\circ} \mathrm{C}$ ?

$$
\mathrm{H}_{2}(g)+\mathrm{I}_{2}(g) \rightleftharpoons 2 \mathrm{HI}(g) \quad K_{\mathrm{c}}=54.5 \text { at } 425.4^{\circ} \mathrm{C} .
$$

5. Hydrogen is industrially synthesized by reacting methane with steam at high temperatures.

$$
\mathrm{CH}_{4}(g)+\mathrm{H}_{2} \mathrm{O}(g) \rightleftharpoons \mathrm{CO}(g)+3 \mathrm{H}_{2}(g)
$$

The value of $K_{\mathrm{c}}$ for the above reaction is 0.0267 at 1133 K . What is $K_{\mathrm{p}}$ for this reaction at 1133 K ?

Stoichiometric coefficients are arbitrary. For example, the oxidation of sulfur dioxide to sulfur trioxide may written as eqn 3. Corresponding to eqn 3, the law of mass action is shown as eqn 3a.

$$
\begin{equation*}
2 \mathrm{SO}_{2}(g)+\mathrm{O}_{2}(g) \rightleftharpoons 2 \mathrm{SO}_{3}(g) \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
K_{p}=\frac{P_{S_{3}}^{2}}{P_{S O_{2}}^{2} P_{O_{2}}} \tag{3a}
\end{equation*}
$$

$$
K_{\mathrm{p}}=0.0012 \text { at } 1500 \mathrm{~K} .
$$

## Key Questions

6. Alternatively, the reaction (eqn $\mathbf{3}$ in the Model) could be written as eqn 4.

$$
\begin{equation*}
\mathrm{SO}_{2}(g)+\frac{1}{2} \mathrm{O}_{2}(g) \rightleftharpoons \mathrm{SO}_{3}(g) \tag{4}
\end{equation*}
$$

a. Complete the law of mass action for eqn 4. (The apostrophe on the $K_{\mathrm{p}}$ (together called " $K_{\mathrm{p}}$-prime") simply tells the reader that this equilibrium constant applies to the same chemical system, but its value is different than the one without an apostrophe.)

$$
K_{p}^{\prime}=
$$

b. Compare eqn $\mathbf{3 a}$ (in the Model) to eqn $\mathbf{4 b}$ (your answer to Question 6a). What must the value of $x$ be in the following equation to make it true?

$$
K_{\mathrm{p}}^{\prime}=\left(K_{\mathrm{p}}\right)^{x}
$$

c. By what factor are the stoichiometric coefficients multiplied in going from eqn 3 to eq n 4 (both of which describe the oxidation of $\mathrm{SO}_{2}$ )? (Compare your answers to Questions 6 b and 6 c .)
7. Given the following generic reversible reaction: $\quad a \mathrm{~A}+b \mathrm{~B} \rightleftharpoons d \mathrm{D}+e \mathrm{E} \quad \boldsymbol{K}$ Suppose all of the stoichiometric coefficients are multiplied by a constant $x$ :

$$
a x \mathrm{~A}+b x \mathrm{~B} \rightleftharpoons d x \mathrm{D}+e x \mathrm{E} \quad \boldsymbol{K}^{\prime}
$$

How is $\boldsymbol{K}^{\prime}$ related to $\boldsymbol{K}$ ?

## Exercise

8. What is the value of $K_{\mathrm{p}}{ }^{\prime}$ at 1500 K for eqn 4 ?

## Key Questions

9. Instead of its formation, suppose we consider the decomposition of sulfur trioxide (eqn 5).

$$
\begin{equation*}
2 \mathrm{SO}_{3}(g) \rightleftharpoons 2 \mathrm{SO}_{2}(g)+\mathrm{O}_{2}(g) \tag{5}
\end{equation*}
$$

a. Finish the law of mass action for eqn 5.

$$
K_{p}^{\prime \prime}=
$$

b. How does the law of mass action for eqn 5 (your answer to Question 9a) compare to the law of mass action for eqn 3 (eqn 3a in the Model)?

## Exercises

10. a. What is the value of $K_{\mathrm{p}}{ }^{\prime \prime}$ for eqn 5 at 1500 K ?
b. A mixture of $\mathrm{SO}_{3}, \mathrm{SO}_{2}$, and $\mathrm{O}_{2}$ gases at 1500 K is found to have the following partial pressures at equilibrium: $P_{\mathrm{SO} 3}=0.013 \mathrm{~atm}$ and $P_{\mathrm{O} 2}=0.788 \mathrm{~atm}$. What is the partial pressure of $\mathrm{SO}_{2}(\mathrm{in} \mathrm{atm})$ ?
11. The $K_{\mathrm{p}}$ for the decomposition of ammonia into its elements $\mathrm{NH}_{3}(g) \rightleftharpoons 3 / 2 \mathrm{H}_{2}(g)+\frac{1}{2} \mathrm{~N}_{2}(g)$ is 111 at $427^{\circ} \mathrm{C}$. What is the value of $K_{\mathrm{p}}$ ' for the following reaction at $427^{\circ} \mathrm{C}$ ?

$$
3 \mathrm{H}_{2}(g)+\mathrm{N}_{2}(g) \rightleftharpoons 2 \mathrm{NH}_{3}(g)
$$

