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## ALE 21. Gibbs Free Energy

(Reference: 20.3 Silberberg $5^{\text {th }}$ edition)

## At what temperature does the spontaneity of a reaction change?

## The Model: The Definition of Free Energy

So far in our study of thermodynamics, we have seen that in order to determine if a reaction is spontaneous we must consider the entropy change $(\Delta \mathrm{S})$ and the enthalpy change $(\Delta \mathrm{H})$ associated with the reaction. An equation that describes the quantitative relationship between entropy and the enthalpy was developed by $\mathbf{J}$. Willard Gibbs, a professor of mathematical physics at Yale in the nineteenth century. He defined a new quantity, now called the Gibbs free energy (G), a single quantity that combines both of these ideas (eqn 1).

$$
\begin{equation*}
G=H-T S \tag{1}
\end{equation*}
$$

Because enthalpy is a state function (i.e., $\Delta H$ is the same between two states, regardless of the path taken getting from the initial to the final state) and entropy is a state function, Gibbs Free Energy is a state function, too.
Just as it was for enthalpies, we often cannot determine the absolute value of the amount of Gibbs free energy that a system has. But that is okay, because we are principally interested in how much work we can obtain from a spontaneous reaction.


The maximum amount of non-expansion work obtainable is equal to $\Delta \mathbf{G}_{r x n}$, and the change in Gibbs free energy for a chemical reaction is something that can be determined.

- T is the temperature in Kelvin

$$
\begin{equation*}
\Delta G=\Delta H-T \Delta S \tag{2}
\end{equation*}
$$

- $\Delta \mathrm{H}$ is $\Delta \mathrm{H}_{\text {reaction }}$ (in Joules or kilojoules)
- $\Delta \mathrm{S}$ is $\Delta \mathrm{S}_{\text {reaction }}$ (in Joules or kilojoules)
- Note: $\Delta \mathrm{H}_{\text {reaction }}$ and $\Delta \mathrm{S}_{\text {reaction }}$ must either both be in J or both in kJ


## Key Questions

1 a. Explain why the Gibbs free energy function (eqn 1) is a state function.
b. Since enthalpy is a state function, the standard enthalpy change of a chemical reaction $\left(\Delta H_{r x n}^{o}\right)$ can be calculated from the standard heats of formation $\left(\Delta H_{f}^{o}\right)$ of the reactants and products. Likewise, since Gibbs free energy is a state function, there's a similar relationship that allows us calculate ( $\Delta G_{r x n}^{o}$ ) from tabulated values of standard molar Gibbs free energies of formation $\left(\Delta G_{f}^{o}\right)$. Write this relationship by completing the equation below.

$$
\Delta G_{r x n}^{o}=
$$

2. The absolute temperature is always positive. But the signs of $\Delta H_{\mathrm{rxn}}$ and $\Delta S_{\mathrm{rxn}}$ may be either negative or positive. Therefore, the sign of $\Delta G_{\mathrm{rxn}}$ may be either positive or negative. Use eqn $\mathbf{2}, \Delta \mathbf{G}=\Delta \mathbf{H}-\mathbf{T} \Delta \mathbf{S}$, to complete the table by filling in each blank with a " $<$ " or a " $>$ ", indicating what the sign of $\Delta G_{\mathrm{rxn}}$ will be in each case.

|  | $\Delta H_{\mathrm{rxn}}<0$ | $\Delta H_{\mathrm{rxn}}>0$ |
| :---: | :---: | :---: |
| $\Delta S_{r x n}<0$ | $\Delta G_{\text {rxn }} \quad 0 \quad 0$ at low $T$ | $\Delta G_{\text {rxn }} \ldots 0$ at low $T$ |
|  | $\Delta G_{\mathrm{rxn}} \ldots \ldots$ at high $T$ | $\Delta G_{\text {rxn }} \ldots \ldots 0$ at high $T$ |
| $\Delta S_{r x n}>0$ | $\Delta G_{\text {rxn }}$ | $\Delta G_{\text {rxn }} \ldots \ldots 0$ at low $T$ |
|  | $\Delta G_{\text {rxn }} \ldots \ldots 0$ at high $T$ | $\Delta G_{\text {rxn }} \ldots \ldots 0$ at high $T$ |

3 a. From the previous question and what we learned in the last ALE, it would seem that $\Delta G_{\mathrm{rxn}}$ has to do with the spontaneity of a chemical reaction. Explain how the sign of $\Delta G_{\mathrm{rxn}}$ can be used to tell whether a chemical reaction is spontaneous or not.
b. What does it mean if $\Delta G_{\mathrm{rxn}}=0$ ? (Hint: Recall that most chemical reactions are reversible.)

## Exercises

4. The standard heats of formation and entropies of diamond and graphite at $25^{\circ} \mathrm{C}$ are given in the table to the right.

|  | $\Delta H_{f}^{o}(\mathrm{~kJ} / \mathrm{mol})$ | $S^{\circ}\left(\mathrm{J} \mathrm{mol}^{-1} \mathrm{~K}^{-1}\right)$ |
| :--- | :---: | :---: |
| C (diamond) | 1.9 | 2.4 |
| C(graphite) | 0 | 5.7 |

Calculate $\Delta H_{r x n}^{o}, \Delta S_{r x n}^{o}$ and $\Delta G_{r x n}^{o}$ for the conversion of diamond to graphite (eqn 3) at $25^{\circ} \mathrm{C}$. Show your work using units and sig. figs. Circle your answers.

$$
\begin{equation*}
\mathrm{C} \text { (diamond) } \rightarrow \mathrm{C} \text { (graphite) } \tag{3}
\end{equation*}
$$

a. $\Delta H_{r x n}^{o}=$
b. $\Delta S_{r x n}^{o}=$
c. $\Delta G_{r x n}^{o}=$
d. What can you say about the slogan "a diamond is forever"?
5. Even at $25^{\circ} \mathrm{C}$, solid iodine sublimes. Gaseous iodine reacts with excess fluorine to form iodine pentafluoride according to the thermochemical equation 4.

$$
\begin{equation*}
\mathrm{I}_{2}(\mathrm{~g})+5 \mathrm{~F}_{2}(\mathrm{~g}) \rightarrow 2 \mathrm{IF}_{5}(\mathrm{~g}) \Delta H^{\circ}=-1743.1 \mathrm{~kJ} \tag{4}
\end{equation*}
$$

The standard Gibbs free energies of formation and standard molar entropies of the reactants and products are provided in the table to the right From this information, determine the standard molar entropy of gaseous iodine at $25^{\circ} \mathrm{C}$.

|  | $\Delta G_{f}^{o}$ <br> $(\mathrm{~kJ} / \mathrm{mol})$ | $S^{\circ}\left(\mathrm{J} \mathrm{mol}^{-1} \mathrm{~K}^{-1}\right)$ |
| :--- | :---: | :---: |
| $\mathrm{I}_{2}(g)$ | 19.4 | $? ? ?$ |
| $\mathrm{~F}_{2}(g)$ | 0 | 203.3 |
| $\mathrm{IF}_{5}(g)$ | -771.6 | 334.5 |

(Hint: You will need to use the version of Hess's law for both $\Delta G_{r x n}^{o}$ and $\Delta S_{r x n}^{o}$.)

## Key Questions

6. Unless $\Delta H_{\mathrm{rxn}}$ and $\Delta S_{\mathrm{rxn}}$ have different signs, the spontaneity of a reaction is temperature dependent. The spontaneity of a chemical reaction changes when $\Delta G_{\mathrm{rxn}}=0$. Hence, if we know $\Delta H_{\mathrm{rxn}}$ and $\Delta S_{\mathrm{rxn}}$ we can use the following equation to determine the approximate temperature at which the spontaneity of a reaction changes (either from spontaneous to non-spontaneous, or from non-spontaneous to spontaneous).

$$
\Delta G_{\mathrm{rxn}}=\Delta H_{\mathrm{rxn}}-T \Delta S_{\mathrm{rxn}}
$$

Starting with the above equation, solve for $T$ (in terms of $\Delta H_{\mathrm{rxn}}$ and $\Delta S_{\mathrm{rxn}}$ ) when $\Delta G_{\mathrm{rxn}}=0$.

## Exercises

7. The standard heats of formation and entropies of liquid water and water vapor at $25^{\circ} \mathrm{C}$ are given in the following table to the right
a. From this data (which are valid at $25^{\circ} \mathrm{C}$ ), estimate the temperature at which water spontaneously boils at a pressure of 1 atm .

|  | $\Delta H_{f}^{o}$ <br> $(\mathrm{~kJ} / \mathrm{mol})$ | $S^{\circ}\left(\mathrm{J} \mathrm{mol}^{-1} \mathrm{~K}^{-1}\right)$ |
| :--- | :--- | :---: |
| $\mathrm{H}_{2} \mathrm{O}(l)$ | -285.8 | 69.9 |
| $\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ | -241.8 | 188.7 |

b. Note that the value you calculated in part a is only an estimation. Briefly explain the source of error in the calculation.

## For the next two Questions:

Refer to thermodynamic data on pages A5-A7 of Appendix B of your textbook.
8. The U.S. government now requires automobile fuels to contain a renewable component. The fermentation of glucose from corn to produce ethanol for addition to gasoline is a key process fulfilling this requirement:

$$
\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6(s)} \rightarrow 2 \mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}_{(l)}+2 \mathrm{CO}_{2(g)}
$$

Calculate $\Delta H^{o}, \Delta \mathrm{~S}^{0}$ and $\Delta G^{o}$ for the reaction at $25^{\circ} \mathrm{C}$. Is the spontaneity of the reaction dependent on temperature? Explain.
9. Consider the following combustion of butane gas:

$$
\mathrm{C}_{4} \mathrm{H}_{10(g)}+6.5 \mathrm{O}_{2(g)} \rightarrow 4 \mathrm{CO}_{2(g)}+5 \mathrm{H}_{2} \mathrm{O}_{(g)}
$$

a.) Predict the signs of $\Delta S^{o}$ and $\Delta H^{o}$. Explain
b.) Calculate $\Delta G^{o}$ by two different methods.

Method \#1: Using $\Delta G_{f}{ }^{o}$

Method \#2: Using $\Delta S^{o}$ and $\Delta H^{o}$
10. Is an endothermic reaction more likely to be spontaneous at higher temperatures or lower temperatures? Explain.

