Answer Key for Nuclear Chemistry Worksheet #2: Energies of Nuclear Reactions
Chem 160 – K. Marr

Key Questions

1. Consider the neutron decay of deuterium (eq **1**).

$$^{2}_{1}\text{H} \rightarrow \, ^{1}_{1}\text{H} + \, ^{1}_{0}\text{n} \qquad (\mathbf{1})$$

a. What is the rest mass (in kg, with six significant figures) of the reactant of eq **1**?

$m_{\text{reactant}} = (2.01355 \text{ amu})(1.66054 \times 10^{-27} \text{ kg/amu}) = \mathbf{3.34358 \times 10^{-27}\ kg}$

b. What is the sum of rest masses (in kg, with six significant figures) of the products of eq **1**?

$m_{\text{products}} = [(1.00728 + 1.00867) \text{ amu}](1.66054 \times 10^{-27} \text{ kg/amu})$
$= (2.01595 \text{ amu})(1.66054 \times 10^{-27} \text{ kg/amu}) = \mathbf{3.34757 \times 10^{-27}\ kg}$

c. The **mass defect** (Δm) for a nuclear reaction is simply the sum of the rest masses of the products minus the sum of the rest masses of the reactants.

$$\Delta m = \sum_{\text{p}} m_{\text{products}} - \sum_{\text{r}} m_{\text{reactants}}$$

What is the mass defect (in kg) for the neutron decay of deuterium (eq **1**)?

$\Delta m = 3.34757 \times 10^{-27} \text{ kg} - 3.34358 \times 10^{-27} \text{ kg} = 0.00399 \times 10^{-27} \text{ kg}$
$= \mathbf{3.99 \times 10^{-30}\ kg}$

d. If $E = mc^2$, then the **nuclear binding energy** (ΔE) of a nucleus is the square of the speed of light (c^2) times the difference between the masses of all nucleons and the mass of the nucleus (Δm).

$$\Delta E = (\Delta m)c^2$$

(Recall that 1 J = 1 kg m^2 s^{-2}.) What is the nuclear binding energy of deuterium (in joules per deuteron)?

$\Delta E = (3.99 \times 10^{-30} \text{ kg})(2.998 \times 10^{8} \text{ m/s})^2 = \mathbf{3.58 \times 10^{-13}\ J}$

2.Consider the beta decay of tritium (eq **2**).

$$ {}^{3}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + {}^{0}_{-1}\mathrm{e} \qquad (\mathbf{2}) $$

As you did in Question 1 for the neutron decay of H-2, determine the following for the beta decay of H-3: (a) the rest mass of the reactant (in kg, with six significant figures); (b) the sum of rest masses of the products (in kg, with six significant figures); (c) the mass defect; and (d) the nuclear binding energy of tritium (in joules per trition).

m_{reactant} = (3.01550 amu)(1.66054 × 10^{-27} kg/amu) = **5.00736 × 10^{-27} kg**

m_{products} = [(3.01493 + 0.00055) amu](1.66054 × 10^{-27} kg/amu)
= (3.01548 amu)(1.66054 × 10^{-27} kg/amu) = **5.00733 × 10^{-27} kg**

Δm = 5.00733 × 10^{-27} kg – 5.00736 × 10^{-27} kg = -0.00003 × 10^{-27} kg
= **-3 × 10^{-32} kg**

ΔE = (-3 × 10^{-32} kg)(2.998 × 10^{8} m/s)^2 = **-3 × 10^{-15} J**

3. You probably remember the First Law of Thermodynamics as: “Energy can neither be created nor destroyed, but it may be converted from one from to another.” What does it mean for tritium to have a “negative nuclear binding energy”? Where does the “missing” energy go? (*Hint*: Consider what *type* of masses you subtracted when you calculated the mass defect.)

A negative binding energy means (like $\Delta H < 0$) that we’re dealing with an exothermic process. Energy, which was potential energy associated with the rest mass of the trition, is being released. When the electron is ejected from the nucleus, it is no longer at rest – the released energy is converted into the kinetic energy that is now possessed by the ejected electron.

4. Why is H-3 radioactive while H-2 is not?

The deuteron has positive nuclear binding energy while the trition has negative binding energy. If the nuclear binding energy is positive, that means that the forces within the nucleus are strong enough to hold the subatomic particles together. But if the nuclear binding energy is negative, the nucleus is unstable and a subatomic particle must be ejected from the nucleus – the excess energy is converted into kinetic energy and carried away from the nucleus.

5. Use the information provided above in the Model to balance each of the following nuclear equations that describe the spontaneous fission that occurs after a slow neutron collides with a U-235 nucleus.

a. $^{1}_{0}n + ^{235}_{92}U \rightarrow ^{72}_{30}Zn + \underline{^{160}_{62}Sm} + \underline{4}\ ^{1}_{0}n$

b. $^{1}_{0}n + ^{235}_{92}U \rightarrow ^{144}_{55}Cs + \underline{^{90}_{37}Rb} + \underline{2}\ ^{1}_{0}n$

c. $^{1}_{0}n + ^{235}_{92}U \rightarrow ^{87}_{35}Br + \underline{^{146}_{57}La} + \underline{3}\ ^{1}_{0}n$

d. $^{1}_{0}n + ^{235}_{92}U \rightarrow ^{140}_{56}Ba + \underline{^{93}_{36}Kr} + \underline{3}\ ^{1}_{0}n$ (**3**)

6. Consider the nuclear reaction in eq **3** (*i.e.*, the reaction of Question 5d). The isotopic masses of U-235, Ba-140, and Kr-93 are 235.04392 amu, 139.91058 amu and 92.93113 amu, respectively.

a. What are the sums of rest masses (in kg, with 6 sig figs) of reactants and products in eq **3**?

$m_{reactants}$ = [(1.00867 + 235.04392) amu](1.66054 × 10^{-27} kg/amu)
= (236.05259 amu)(1.66054 × 10^{-27} kg/amu) = **3.91975 × 10^{-25} kg**

$m_{products}$ = {[139.91058 + 92.93113 + 3(1.00867)] amu}(1.66054 × 10^{-27} kg/amu)
= (235.86772 amu)(1.66054 × 10^{-27} kg/amu) = **3.91668 × 10^{-25} kg**

b. What is the mass defect (in kg) for the absorption of a neutron by U-235 followed by spontaneous fission?

Δm = 3.91668 × 10^{-25} kg – 3.91975 × 10^{-25} kg = -0.00307 × 10^{-25} kg
= **-3.07 × 10^{-28} kg**

c. How much energy (in kJ) is released *per mole* of U-235 that is consumed as fuel? (Yes, nuclear fission reactors have their problems – the build-up of radioactive “left-over” by-products of the fission reactions, plus the possibility of going critical. But to understand why fission reactors are economical to use, compare the amount of energy you just calculated to the $\Delta G°$’s of -5300 kJ/mol for the combustion of octane and -474 kJ/mol for the cell reaction in the hydrogen-oxygen fuel cell.)

ΔE = (-3.07 × 10^{-28} kg / U-235 nucleon)(2.998 × 10^{8} m/s)^2
ΔE = (-2.76 × 10^{-11} J / U-235 nucleon)(6.022 × 10^{23} U-235 nucleons / 1 mol U-235)
ΔE = (-1.66 × 10^{13} J/mol U-235)(1 kJ / 10^{3} J)
ΔE = **-1.66 × 10^{10} kJ/mol U-235** (*i.e.*, For every mole of U-235 that is consumed, 16.6 terajoules of energy are released!)