Useful Equations: Physics 222

ENERGY:

Kinetic Energy: $K = \frac{1}{2} m v^2$ $\left(1\text{ J} = 1\frac{kg\, m^2}{s^2}\right)$

Work: work $= \int \vec{F} \cdot d\vec{x} = \vec{F} \cdot \Delta\vec{x}$ (if force is constant)

Work-energy theorem: Net work $= \int \left(\Sigma\vec{F}\right) \cdot d\vec{x} = \Delta K$

CONSERVATIVE FORCES:

Let $\vec{F}_U = F_{U,x}\,\hat{i} + F_{U,y}\,\hat{j} + F_{U,z}\,\hat{k}$

Then $\vec{F}_{U,x} = -\frac{dU}{dx}$, $\vec{F}_{U,y} = -\frac{dU}{dy}$, and $\vec{F}_{U,z} = -\frac{dU}{dz}$

And $\Delta U = -\int \vec{F}_U \cdot d\vec{x}$

FLUIDS: Flow rate: $flow\ rate = \frac{volume}{time} = velocity \cdot Area$ or $\int v\, dA$

Buoyancy: $F_{Buoyancy} = \rho_{fluid}\, g\, V$ where V is the volume displaced

Static Fluid Pressure: $P = \rho\, g\, depth$ if ρ is constant

Fluid Potential: $(Fluid\ potential) = \Psi = P + \rho g y$ if ρ is constant

Bernoulli's equation (in the absence of friction):

$$(Fluid\ potential) + \frac{K}{Volume} = C \quad \text{or} \quad P + \rho\, g\, y + \tfrac{1}{2}\rho v^2 = \text{constant}$$

Viscosity: $\eta = \frac{F\, d}{A\, v}$ kinematic viscosity $= \mu = \frac{\eta}{\rho}$

Viscosity controlled flow: $(potential\ difference) = (flow\ rate) \times (\text{resistance})$

Poisuille's equation (hollow pipes): resistance$= \frac{8\,\eta\,\ell}{\pi\, r^4}$ $(\ell = length,\ r = radius)$

Darcy's equation (flow through materials):

$$resistance = \frac{\eta\,\ell}{\kappa\ Area} \quad (\ell = length, \kappa = permeability)$$

<u>ELECTRIC FORCES, FIELDS, AND POTENTIALS</u>:

Coulomb's Law and Electric Fields:
The magnitude of the force between point charges separated by a distance r is given by

$$F = \frac{k\, q_1\, q_2}{r^2} = \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{q_1\, q_2}{r^2}$$

$$k = 8.99 \times 10^9 \ \frac{N\, m^2}{C^2} \qquad \varepsilon_0 = 8.854 \times 10^{-12} \ \frac{C^2}{N\, m^2}$$

The electric field is the force per unit charge, $\vec{F} = q\left[\vec{E}\right]$.
For a point charge the magnitude is given by

$$E = \frac{k\, q}{r^2}$$

Electric Potential:
The electric potential V is the potential energy per unit charge:

$$V = -\int \vec{E} \cdot d\vec{x}\,, \qquad \text{and} \qquad E_x = -\frac{dV}{dx}, \quad E_y = -\frac{dV}{dy}, \quad \text{etc.}$$

If we choose to set the potential of points infinitely far from charges to be zero, then the potential of a point charge is given by:

$$V = \frac{k\, q}{r} = \left(\frac{1}{4\pi\varepsilon_0}\right) \frac{q}{r}$$

Gauss' Law: The flux of the electric field through a closed surface is proportional to the charge enclosed. $\Phi_E = \frac{1}{\varepsilon_0} Q_{inside} = 4\pi k\ Q_{inside}$

<u>ELECTRIC CIRCUITS</u>:

Resistors (Ohm's Law): $\Delta V = IR$ **Power:** $\text{Power} = \Delta V\, I = I^2 R$

Capacitors: $Q = C\,\Delta V$ **Parallel Plate Capacitors:** $C = \frac{\varepsilon\, A}{d}$ $\varepsilon = \kappa\,\varepsilon_0,\ \ \kappa > 1$

Kirchhoff's Rules:

1. **"Current in = current out" or "**$\sum I_{in} = 0$**"**
2. **Going around a closed loop,** $\sum voltages = 0$

Resistance: $R = \frac{\rho\, L}{A}$ $\rho = resistivity$ $conductivity = \sigma = \frac{1}{\rho}$

In series *currents* are equal. In parallel, *voltages* are equal.

Resistors in parallel: $\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{\text{TOTAL}}}$ **…in series:** $R_1 + R_2 = R_{\text{TOTAL}}$

Capacitors in parallel: $C_1 + C_2 = C_{\text{TOTAL}}$ **… in series:** $\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{\text{TOTAL}}}$