Useful Equations: Physics 222

FORCE and MOMENTUM:

Momentum: $\vec{p} = m\vec{v}$ (and $\vec{p}$ is conserved in the absence of external forces)

Force: **2nd law:** $\sum \vec{F} = \frac{d\vec{p}}{dt}$ *or* $\sum \vec{F} = m\vec{a}$ (if mass is constant)

3rd law: $\vec{F}_{AB} = -\vec{F}_{BA}$

Impulse: $\text{Impulse} = \int \vec{F}\, dt$ and $\int \Sigma\vec{F}\, dt = \Delta\vec{p}$

ENERGY:

Kinetic Energy: $K = \frac{1}{2}mv^2$ $\left(1\text{ J} = 1\frac{kg\, m^2}{s^2}\right)$

Work: $\text{work} = \int \vec{F}\cdot d\vec{x} = \vec{F}\cdot\Delta\vec{x}$ (if force is constant)

Work-energy theorem: $\text{Net work} = \int \left(\Sigma\vec{F}\right)\cdot d\vec{x} = \Delta K$

Gravity: $\vec{F} = m\vec{g}$ (pointing down), $U = mgh$

Ideal Springs: $\vec{F} = -k\vec{x}$, $U = \frac{1}{2}kx^2$

CONSERVATIVE FORCES:

Let $\vec{F}_U = F_{U,x}\,\hat{i} + F_{U,y}\,\hat{j} + F_{U,z}\,\hat{k}$

Then $\vec{F}_{U,x} = -\frac{dU}{dx}$, $\vec{F}_{U,y} = -\frac{dU}{dy}$, and $\vec{F}_{U,z} = -\frac{dU}{dz}$

And $\Delta U = -\int \vec{F}_U \cdot d\vec{x}$

ROTATION:

If angular acceleration is constant,

$$\Delta\vec{\theta} = \vec{\omega}_0 t + \tfrac{1}{2}\vec{\alpha}\, t^2 \quad , \quad \vec{\omega}(t) = \vec{\omega}_0 + \vec{\alpha}\, t$$

$$\Delta\vec{\theta} = \tfrac{1}{2}\left(\vec{\omega}_0 + \vec{\omega}\right)t \qquad \tfrac{1}{2}\omega_f^2 - \tfrac{1}{2}\omega_i^2 = \vec{\alpha}\cdot\left(\Delta\vec{\theta}\right)$$

Rotational Inertia: $I = \sum m\, r^2$ and for a rolling object $I = \beta\, M\, R^2$

Parallel axis theorem: $I_{about\ any\ point} = I_{about\ the\ CM} + MR_{CM}^2$

Kinetic Energy: $K_{translation} = \tfrac{1}{2} m\, v^2$ and $K_{rotation} = \tfrac{1}{2} I\, \omega^2$

Work: $Work = \int \tau\, d\theta$ or $Work = \tau\, \Delta\theta$ if torque is constant

Angular momentum: $\vec{\ell} = I\, \vec{\omega}$ (for a rigid body) or $\vec{\ell} = \vec{r} \times \vec{p}$ (always)

Torque: $\vec{\tau} = \vec{r} \times \vec{F}$

Newton's second law: $\begin{cases} \Sigma\vec{\tau} = I\, \vec{\alpha} & \text{if } I \text{ is constant} \\ \Sigma\vec{\tau} = \dfrac{d\vec{\ell}}{dt} = \dfrac{d}{dt}\left(\vec{r}\times\vec{p}\right) & \text{(always)} \end{cases}$

FLUIDS:

Flow rate: $flow\ rate = \dfrac{volume}{time} = velocity \cdot Area$ or $\int v\, dA$

Static Fluid Pressure: $P = \rho\, g\, depth$ if ρ is constant

Fluid Potential: $fluid\ \ potential = \mathbf{P} = \dfrac{U}{vol.} = P + \rho\, g\, y$ if ρ is constant

Bernoulli's equation (in the absence of friction):

$$\mathbf{P} + \frac{K}{vol.} = \text{constant} \quad \text{OR} \quad P + \rho\, g\, y + \tfrac{1}{2}\rho\, v^2 = \text{constant}$$

Viscosity: $\eta = \dfrac{F\, d}{A\, v}$ kinematic viscosity $= \mu = \dfrac{\eta}{\rho}$

Viscosity controlled flow: $\left(potential\ difference\right) = \left(flow\ rate\right) \times \left(\text{resistance}\right)$

Poisuille's equation (hollow pipes): Resistance $= \dfrac{8\,\eta\, L}{\pi\, r^4} = \dfrac{8\pi\,\eta\, L}{A^2}$
$(L = length,\ \ r = radius,\ \ A = area)$

Darcy's equation (flow through materials):
Resistance $= \dfrac{\eta\, L}{\kappa\, A}$ $\quad (L = length,\ \ \kappa = permeability,\ \ A = area)$

ELECTRIC FORCES, FIELDS, AND POTENTIALS:

Coulomb's Law and Electric Fields:
The magnitude of the force between point charges separated by a distance *r* is given by

$$F = \frac{k\, q_1\, q_2}{r^2} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1\, q_2}{r^2}$$

The electric field is the force per unit charge, $\vec{F} = q\left[\vec{E}\right]$.
For a point charge the magnitude is given by

$$E = \frac{k\, q}{r^2}$$

Electric Potential:
The electric potential *V* is the potential energy per unit charge:

$$V = -\int \vec{E}\cdot d\vec{x}\,, \qquad \text{and} \qquad E_x = -\frac{dV}{dx},\quad E_y = -\frac{dV}{dy},\quad \text{etc.}$$

If we choose to set the potential of points infinitely far from charges to be zero, then the potential of a point charge is given by:

$$V = \frac{k\, q}{r} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{r}$$

ELECTRIC CIRCUITS:

Resistors (Ohm's Law): $\Delta V = IR$ **Power:** $\text{Power} = \Delta V\, I = I^2 R$

Kirchhoff's Rules:

1. **"Current in = current out" or "**$\sum I_{in} = 0$**"**
2. **Going around a closed loop,** $\sum voltages = 0$

Resistors in parallel: $\frac{1}{R_{\text{TOTAL}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$ **...in series:**

$$R_{\text{TOTAL}} = R_1 + R_2 + \cdots$$