Physics 222
Homework Assignment #1
Review of mechanics and the Fundamental Theorem of Calculus

1. This is a pure math problem about some special functions but you don't need to know any more about the functions than what is given here. You also shouldn't need to reach for Maple, MathCAD, or Derive to solve these. An ordinary scientific calculator and the fundamental theorem of calculus should be enough.

   The Gamma function $\Gamma(x)$ is a continuous and differentiable function for all positive values of *x*. It has the nifty property that for integer values of *x*, $\Gamma(x) = (x-1)!$ (that's a factorial symbol, not a statement that I'm excited). So $\Gamma(4) = 3! = 6, \Gamma(5) = 4! = 24,$ and so on.

   The Digamma function $\psi(x)$ is a continuous function defined as

   $$\psi(x) = \frac{d}{dx} \ln[\,\Gamma(x)]$$

   (which is probably just what you thought it would be).

   Using only a pocket calculator and the fundamental theorem of calculus, evaluate:

   $$\int_{7.0}^{11.0} \psi(x)\, dx$$

2. Another pure math problem. The logarithmic integral function $Li(x)$ is a continuous and differentiable function defined as

   $$Li(x) = \int_2^x \frac{1}{\ln(y)}\, dy$$

   Using only a pocket calculator and the fundamental theorem of calculus, find the slope of the function $Li(x)$ when *x = 2.3* (one short calculator calculation should do it).

3. A certain plook is a distance *x* away from a zoit. The potential energy of the plook-zoit system varies with the distance *x* and is given by

   $$U(x) = a + b\,x + c\,x^2$$

   Where *a = 1.73 joules, b = 3.52 joules/meter, and c = 11.2 joules/meter*^{2}.

   a) Is the plook attracted to the zoit or repelled by it? Explain your reasoning.
   b) What is the magnitude of the force between them when they are separated by a distance of 0.20 meters?
   c) What piece of important-looking information in this problem is completely unnecessary to answer questions a and b?
   d) Why is that piece of information useless? Explain in words.

4. A toy car begins from rest and is then subjected to a net force in the positive direction which increases at a steady rate from zero to 2.4 newtons in a period of 3.0 seconds. It then decreases back to zero at a steady rate in 1.0 seconds.
    a. What was the momentum of the toy car at the end of those 4.0 seconds?
    b. The final kinetic energy of the toy car was 14.4 joules. What was the mass of the car?

5. A book is sliding across the floor under the influence of kinetic friction (which is constant). When the book starts sliding it has a speed of 7.0 m/s. It slows down as it travels, so when it reaches the far wall it has a speed of 1.0 m/s.
    a. State which happens first and explain your reasoning:
        i. the book reaches the point halfway across the room (in terms of distance) or
        ii. the book completes half of the journey in terms of time taken?
    b. What's the speed of the book when it has completed half of the journey in terms of time taken?
    c. What's the speed of the book when it has competed half of the journey in terms of distance traveled (when it is halfway across the floor)?

    (No you don't need to know the mass of the book or the magnitude of the friction force to solve problem #5. You only need to know that the net force on the book was constant.)