Useful Equations: Physics 222

FORCE and MOMENTUM:

Momentum: $\vec{p} = m\vec{v}$ (and $\vec{p}$ is conserved in the absence of external forces)

2nd law: $\sum \vec{F} = \frac{d\vec{p}}{dt}$ *or* $\sum \vec{F} = m\vec{a}$ (if mass is constant) **3rd law:** $\vec{F}_{AB} = -\vec{F}_{BA}$

Impulse: $\text{Impulse} = \int \vec{F}\, dt$ and $\int \Sigma\vec{F}\, dt = \Delta\vec{p}$

ENERGY: Kinetic Energy:

$K = \frac{1}{2}mv^2$ $\left(1\ \text{J} = 1\frac{kg\ m^2}{s^2}\right)$

Work: $\text{work} = \int \vec{F}\cdot d\vec{x} = \vec{F}\cdot\Delta\vec{x}$ (if force is constant)

Work-energy theorem: Net work $= \int \left(\Sigma\vec{F}\right)\cdot d\vec{x} = \Delta K$

Gravity: $\vec{F} = m\vec{g}$ (pointing down), $U = mgh$

Ideal Springs: $\vec{F} = -k\vec{x}$, $U = \frac{1}{2}kx^2$

CONSERVATIVE FORCES:

Let $\vec{F}_U = F_{U,x}\,\hat{i} + F_{U,y}\,\hat{j} + F_{U,z}\,\hat{k}$

Then $\vec{F}_{U,x} = -\frac{dU}{dx}$, $\vec{F}_{U,y} = -\frac{dU}{dy}$, and $\vec{F}_{U,z} = -\frac{dU}{dz}$, And $\Delta U = -\int \vec{F}_U \cdot d\vec{x}$

ROTATION:

If angular acceleration is constant,

$$\Delta\vec{\theta} = \vec{\omega}_0 t + \tfrac{1}{2}\vec{\alpha}\,t^2 \quad , \quad \vec{\omega}(t) = \vec{\omega}_0 + \vec{\alpha}\,t$$

$$\Delta\vec{\theta} = \tfrac{1}{2}\left(\vec{\omega}_0 + \vec{\omega}\right)t \qquad \tfrac{1}{2}\omega_f^2 - \tfrac{1}{2}\omega_i^2 = \vec{\alpha}\cdot\left(\Delta\vec{\theta}\right)$$

Rotational Inertia: $I = \sum m\,r^2$ and for a rolling object $I = \beta\, M\, R^2$

Kinetic Energy: $K_{translation} = \frac{1}{2}m\,v^2$ and $K_{rotation} = \frac{1}{2}\,I\,\omega^2$

Work: $Work = \int \tau\, d\theta$ or $Work = \tau\,\Delta\theta$ if torque is constant

Angular momentum: $\vec{\ell} = I\,\vec{\omega}$ (for a rigid body) or $\vec{\ell} = \vec{r}\times\vec{p}$ (always)

Torque: $\vec{\tau} = \vec{r}\times\vec{F}$

Newton's second law:
$$\begin{cases} \Sigma\vec{\tau} = I\,\vec{\alpha} & \text{if } I \text{ is constant} \\ \Sigma\vec{\tau} = \dfrac{d\vec{\ell}}{dt} = \dfrac{d}{dt}\left(\vec{r}\times\vec{p}\right) & \text{(always)} \end{cases}$$

Fluid Equations

Pressure: $P = \frac{Force}{Area}$ SI unit: $1\,pascal = 1\,Pa = 1\frac{N}{m^2}$

Fluid Statics: $P = \rho\ g\ Depth$ $\rho = density$ $Depth = Depth$

Incompressible fluid flow:

$flow\ rate = \frac{volume}{time} = (area)(velocity)$

In the absence of sources or sinks: $flow\ in = flow\ out$

Fluid Potential:

For any kind of potential: $Potential = V \equiv \frac{Potential\ energy}{unit\ of\ stuff}$

$Fluid\ Potential = V_f = \frac{Potential\ energy}{volume}$ $V_f = P + \rho g y$

Bernoulli's equation:

$$P + \rho g y + \frac{1}{2}\rho v^2 = constant$$

Viscosity: SI unit = Pa·s

$$\eta = \frac{F\,\Delta y}{A\,\Delta v} \quad or \quad \frac{dv}{dy} = \frac{F}{\eta\,A}$$

Poiseuille's Equation:

$flow\ rate = (\Delta V_f)\,(C_f)$ $C_f = fluid\ conductance\ \ in\ \ \frac{m^3}{Pa \cdot s}$

In a cylindrical tube: $C_f = \frac{\pi\,R^4}{8\,\eta\,L}$ $R = radius$ $L = length$