Thin film interference

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Calculation of thin film interference:

A ray of light (traveling through material with an index of refraction n_1) strikes a flat sheet of material (with index of refraction n_2) as shown in Figure 1. The sheet of material has thickness T.

As shown in the diagram, the ray of light strikes the material at point A with incident angle θ_1 where it is split into two beams. One of the beams reflects at point A and travels thrhough point B. The other beam refracts through the material at an angle θ_2 where (according to Snell's Law):

$$n_1 \ \sin\theta_1 = n_2 \ \sin\theta_2 \ .$$

This transmitted beam travels through the material to point C where it is partially reflected and re-emerges from the sheet of material at point E, a distance $|AE| = 2\,T\,\tan\theta_2$ from point A.

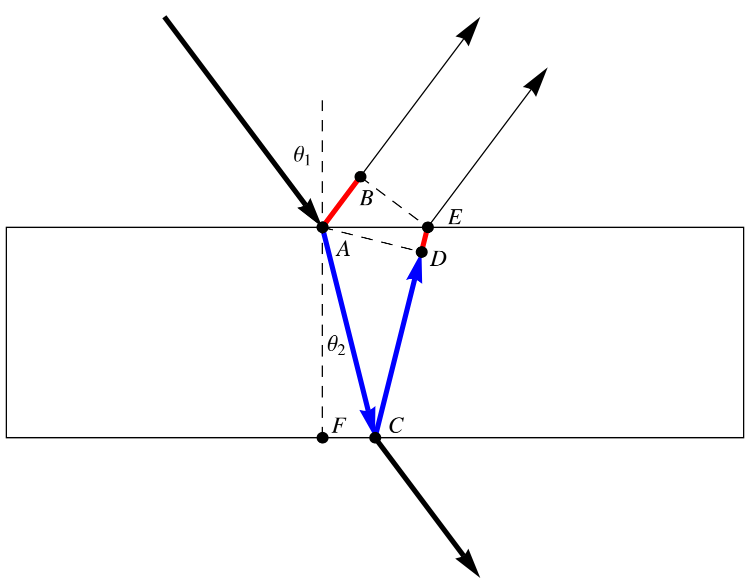

Figure 1: A ray of light originating in a material with index of refraction n_1 is incident on a sheet of material of thickness T (the distance from points A to F) and index of refraction n_2. The light is incident at point A with an angle θ_1. Part of the ray of light reflects back through point B, while part refracts at an angle θ_2 to C, reflects back to point E, and continues outward in a ray parallel to the ray at B. Rays of light passing through points B and E differ in phase because of different path lengths, different wavelengths in the two materials, and inversions upon reflection. The number of waves in the external segment AB is the same as the number of waves inside the material along path DE, so the effective difference in the lengths of the two paths is $n_2\,(|AC| + |CD|)$.

The beams that emerge at points C and E differ in phase for three reasons. First, the path from point A to point B is shorter than the path from point A to C to E. Second the wavelength of light in the material will differ from the wavelength outside (waves per unit of path length differ by n_2/n_1). Third, waves invert their phase upon reflection from a surface of higher index of refraction.

Regardless of indices of refraction, the number of waves that occur along the path from point A to B (outside of the sheet of material) is the same as the number that occur along the path from point D to A (inside the material). The number of waves (N) along each path can be found from

$$N_{AB} = \frac{|AB|}{\lambda_1} = \frac{|AE|\,\sin\theta_1}{\lambda_1} = \frac{|AE|\left[\left(\frac{n_2}{n_1}\right)\sin\theta_2\right]}{\left(\frac{n_2}{n_1}\right)\lambda_2} = \frac{|AE|\,\sin\theta_2}{\lambda_2} = \frac{|DE|}{\lambda_2} = N_{DE}\ .$$

The difference in phase between the beams at B and E is thus due entirely to waves that lie along the path from A to C to D. As seen in Figure 2, the length of that path is $|AC|+|CD| = 2\,T\,\cos\theta_2$.

There will be an inversion of phase upon each reflection off of a surface of higher index of refraction, which could happen at the upper surface, the lower surface, or both. This potentially adds half of a wave to the phase difference between the beams. Intereference of the emerging beams is thus determined by the number of waves (N) from A to D and the number of inversions of phase:

$$2\,T\,\cos\theta_2 = N\,\lambda_2 \quad , \quad \text{or} \quad 2\,T\sqrt{{n_2}^2 - (n_1\,\sin\theta_1)^2} = N\,n_1\,\lambda_1 .$$

- If there is a reversal of phase at *neither* or *both* of the two interfaces (as is true for anti-reflective coatings on lenses), then there will be **constructive** interference if if $N = n$, where $n = 1, 2, 3, \ldots$, and there will be **destructive** interference if $N = \left(n + \frac{1}{2}\right)$.
- If there is a reversal of phase at *just one* of the two surfaces (as is true in the case of oil films on water and soap bubbles in air), then there will be **constructive** interference if $N = \left(n + \frac{1}{2}\right)$, where $n = 1, 2, 3, \ldots$, and there will be **destructive** interference if $N = n$.

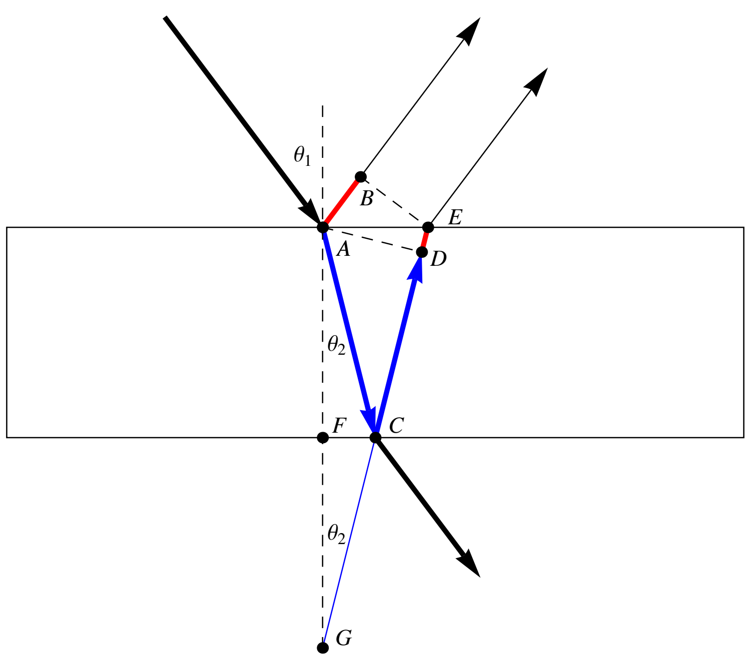

Figure 2: The number of waves is the same in the two red segments: AB and DE. The difference in phase between the emerging beams is due to the blue segments: $|AC|+|CD| = |GD|$. The difference in path length is thus $2\,T\,\cos\theta_2$, and the difference in the number of waves is thus $(2\,T\,\cos\theta_2)/\lambda_2$.