Circuits I: (without inductance):

Resistors (Ohm's Law): $V = IR$ Power: Power $= VI = I^2 R$

Kirchhoff's Rules:

1. *Junctions:* " $\sum I_{in} = \sum I_{out}$ ",   2. Loops: $\sum voltages = 0$

Resistors in parallel: $\dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{1}{R_{TOTAL}}$ ...in series: $R_1 + R_2 = R_{TOTAL}$

Capacitors: $Q = C \, \Delta V$

Capacitors in parallel: $C_1 + C_2 = C_{TOTAL}$ ... in series: $\dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{C_{TOTAL}}$

RC Circuits: Time constant: $\tau = RC$

Discharging or charging: $f(t) = f_1 \, e^{-t/\tau} + f_2$

Often: $f(t) = f_0 \, e^{-t/\tau}$ or $f(t) = f_\infty \left(1 - e^{-t/\tau}\right)$

Magnetism: Forces: $\vec{F}_{magnetic} = q\vec{v} \times \vec{B} = \ell \vec{I} \times \vec{B}$ or $\vec{F}_{EM} = q\left[\vec{E} + \left(\vec{v} \times \vec{B}\right)\right]$

Magnetic fields from currents: A distance r from a straight wire: $\left|\vec{B}\right| = \dfrac{\mu_0}{2\pi}\dfrac{I}{r}$

At the center of a loop of wire of radius R: $\left|\vec{B}\right| = \dfrac{\mu_0}{2}\dfrac{I}{R}$

In a cylindrical coil (or inductor): $\left|\vec{B}\right| = \dfrac{\mu_0 \, N \, I}{length}$

Torque on a loop of wire: $\vec{\tau} = \vec{\mu} \times \vec{B}$ where $\mu = N \, I \, Area$

Inductance: $EMF = -L\dfrac{dI}{dt}$,   and for a cylindrical inductor $\left|\vec{B}\right| = \dfrac{\mu_0 \, N \, I}{length}$

For an LC circuit (capacitor and inductor):

$q = q_{max} \sin\left(\omega_0 t - \phi\right)$, $I = \dot{q}$, $EMF_L = -L\ddot{q}$,   and $\omega_0 = \sqrt{\dfrac{1}{LC}}$

Maxwell's equations: $\epsilon_0 = 8.854 \times 10^{-12} \, C^2/(N \, m^2)$ $\mu_0 = 4\pi \times 10^{-7} \, T \, m/A$

Gauss' law (for $\vec{E}$ and $\vec{B}$) $\int_{\substack{closed \\ surface}} \vec{E} \cdot d\vec{A} = \dfrac{Q}{\epsilon_0}$ **and** $\int_{\substack{closed \\ surface}} B \cdot d\vec{A} = 0$

Faraday's law: $\mathcal{E}mf = -\dfrac{d\Phi_B}{dt}$ or $\int_{loop} \vec{E} \cdot d\vec{\ell} = -\dfrac{d}{dt}\int \vec{B} \cdot d\vec{A}$

Ampere-Maxwell law: $\int_{loop} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{encloed} + \mu_0 \, \epsilon_0 \, \dfrac{d}{dt}\int \vec{E} \cdot d\vec{A}$

Harmonic oscillators:

Angular frequency: $\omega = 2\pi f = \dfrac{2\pi}{T}$

SHO Equation: $\ddot{x} = -\omega^2 x$

Solution to SHO Equation:

$$x - x_{eq} = A\sin(\omega_0 t - \varphi)$$

where ω_0 is the natural *angular* frequency of the oscillator

Mass on a spring: $\omega_0 = \sqrt{\dfrac{k}{m}}$ Simple pendulum: $\omega_0 = \sqrt{\dfrac{g}{\ell}}$

Wave kinematics:

Traveling wave: $f(x,t) = f(x \pm vt)$

Standing wave: $f(x,t) = f(x + vt) \pm f(x - vt)$

Sinusoidal wave: $f(x,t) = A\sin(kx \pm \omega t \pm \varphi)$
or $A\sin(kx + \omega t + \varphi) \pm A\sin(kx - \omega t + \varphi)$

$$k = \frac{2\pi}{\lambda} \quad \omega = \frac{2\pi}{T} \quad v = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k}$$

Wave dynamics:

The wave equation: I forget…

The wave equation is either $\dfrac{\partial^2 f}{dx^2} = v^2 \dfrac{\partial^2 f}{dt^2}$ or $\dfrac{\partial^2 f}{dt^2} = v^2 \dfrac{\partial^2 f}{dx^2}$

I can't remember which it is, but you ought to be able to figure it out.

Velocities:

strings: $v = \sqrt{\dfrac{\tau}{\mu}}$ sound: $v = \sqrt{\dfrac{B}{\rho}} = \sqrt{\dfrac{\gamma P}{\rho}}$ light: $v = c = \sqrt{\dfrac{1}{\mu\varepsilon}}$

Sound:

Intensity and sound level (volume):

Intensity $= I = \dfrac{\text{Power}}{\text{Area}}$ $SI\ Unit = \dfrac{watt}{meter^2} = W\ m^{-2}$

Sound level in decibels $= \beta = 10\log_{10}\left(\dfrac{I}{I_0}\right)$ $\left[I_0 = 10^{-12}\ W/m^2\right]$

Doppler Effect for SOUND:

$$f_{observed} = \left(\frac{v_s \pm v_{observer}}{v_s \mp v_{source}}\right) f_{emitted}$$

Damped and/or driven harmonic oscillators:

Damped springs: if $F_{spring} = -kx$ and $F_{friction} = -bv$

Without a driver (motor):

$$x = x_{max} \, e^{\left(-\frac{bt}{2m}\right)} \sin(\omega t - \phi) \quad \text{where} \quad \omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}, \quad v_{max} = \omega \, x_{max}, \quad \text{etc.}$$

With a driver (motor with angular frequency ω):

$$v_{max} = \frac{(\omega F / m)}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \left(\frac{b\omega}{m}\right)^2}} = \frac{F}{\sqrt{\left(m\omega - \frac{k}{\omega}\right)^2 + (b)^2}}$$

Damped harmonic oscillators and RLC Circuits

Springs with fluid friction: $F_{spring} = -kx$ $F_{friction} = -bv$

Kinematics: $v = \dot{x}$ $a = \dot{v} = \ddot{x}$ and $-kx - bv = ma$

Resonant frequency: $\omega_0 = \sqrt{\frac{k}{m}}$

Damped harmonic oscillation:

$$x = x_{max} \, e^{\left(-\frac{bt}{2m}\right)} sin(\omega t - \varphi), \qquad \omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}, \quad v_{max} = \omega \, x_{max}$$

RLC *series* circuits: $\Delta V_C = \frac{1}{C} q, \quad \Delta V_R = RI, \quad \Delta V_L = L\dot{I}, \quad$ where $\dot{q} = I \quad \dot{I} = \ddot{q}$

And $\frac{1}{C} q + RI + L\dot{I} = 0$

Resonant frequency: $\omega_0 = \frac{1}{\sqrt{LC}}$

Without a driver (AC source):

$$I = I_{max} \, e^{\left(-\frac{Rt}{2L}\right)} \sin(\omega t - \phi) \quad \text{where} \quad \omega = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2}, \quad I_{max} = \omega \, q_{max}, \quad \text{etc.}$$

Driven damped harmonic oscillators and AC Circuits:

With a driver (AC source with angular frequency ω):

$$EMF = V_{max} \sin(\omega t)$$

$$I_{max} = \frac{V_{max}}{Z} = \frac{V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_{max}}{R \sqrt{1 + Q^2 \left(x - \frac{1}{x}\right)^2}}$$

$$= \frac{V_{max}}{R} \cos\varphi$$

Symbols and vocabulary for AC Circuits

Voltage	Reactance	
$\Delta V_L = X_L\, I_{MAX} \cos(\omega t - \varphi)$	$X_L = \omega L$	$\omega = x\,\omega_0$
$\Delta V_R = X_R\, I_{MAX} \sin(\omega t - \varphi)$	$X_R = R$	$Q^2 = \dfrac{L}{R^2\,C}$
$\Delta V_C = -X_C\, I_{MAX} \cos(\omega t - \varphi)$	$X_C = \dfrac{1}{\omega C}$	$\tan\varphi = \dfrac{X_C - X_L}{R}$

Power: $Power_{MAX} = V_{MAX}\, I_{MAX}\ \cos\varphi = \dfrac{(V_{max})^2}{R}\ \cos^2\varphi$

$$Power_{RMS} = V_{RMS}\, I_{RMS}\ \cos\varphi = \frac{1}{2}\ Power_{MAX}$$

Q Factor: for an RLC circuit, $Q^2 = \dfrac{L}{R^2\,C}$

Optics:

$$v_{light\ in\ material} = \frac{c}{n}\ ,\quad n \ge 1\quad \left(c = 3.00 \times 10^8\ m/s = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

Index of refraction:

$$v = \frac{1}{\sqrt{\mu\,\varepsilon}} \approx \frac{1}{\sqrt{\mu_0\,\varepsilon}} = \frac{1}{\sqrt{\mu_0\,(\kappa\,\varepsilon_0)}} = \frac{c}{\sqrt{\kappa}}\ ,\quad \text{so}\quad n \approx \sqrt{\kappa}$$

Reflection: $\theta_{incident} = \theta_{reflected}$ Refraction: $n_1\ \sin\theta_1 = n_2\ \sin\theta_2$

$$Intensity = \frac{Power}{Area}\qquad \text{For waves:}\ I_{RMS} = \frac{1}{2}\,I_{max}$$

Intensity:

$$I_{max} = \left|\frac{1}{\mu_0}\left(\vec{E}_{max} \times \vec{B}_{max}\right)\right| = c\varepsilon_0\,E_{max}^2 = \frac{c}{\mu_0}\,B_{max}^2\ ,\qquad E_{max} = c\,B_{max}$$

Polarization:

When unpolarized light passes through one polarizer: $I_1 = \frac{1}{2}\,I_0$

When polarized light passes through a second polarizer rotated at an angle of θ from the first: $I_2 = I_1 \cos^2\theta = \frac{1}{2}\,I_0\ \cos^2\theta$