

Newton's Law of Cooling

A container of hot water at temperature, T , placed in a room of lower temperature T_{room} , will result in an exchange of heat from the hot water to the room. The water will eventually cool to the same temperature as the room. You observe this cooling process every time you wait for a hot drink to cool. In this experiment you will examine the cooling of hot water, with the goal of creating a model that describes the process. You can also predict the time it takes for the hot water to cool to room temperature.

Isaac Newton modeled the cooling process by assuming that the rate at which thermal energy moved from one body to another is proportional (by a constant k) to the difference in temperature between the two bodies, T_{diff} . In the case of a sample of water cooling in room temperature air

$$\text{cooling rate} = -kT_{diff}$$

From this simple assumption he showed that the temperature change is exponential in time and can be predicted by

$$T_{diff} = T_0 e^{-kt}$$

where T_0 is the initial temperature difference. Exponential changes are common in science. Systems in which a rate of change is proportional to the changing quantity show exponential behavior.

To complete this experiment in a short time, you will use a small quantity of hot water, at a temperature about 30°C above room temperature. A temperature probe will record the water's temperature as it cools.

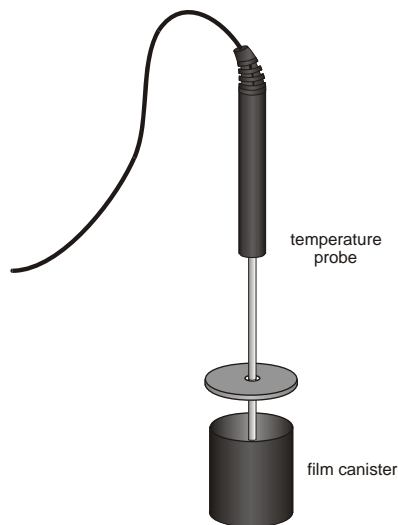


Figure 1

OBJECTIVES

- Use a Temperature Probe to record the cooling process of hot water.
- Test Newton's law of cooling using your collected water temperature data.
- Use Newton's law of cooling to predict the temperature of cooling water at any time.

MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator	35 mm film canister with top
EasyData application	hot water
Temperature Probe and data-collection interface or EasyTemp	

PROCEDURE

1. Turn on the calculator. Connect the Temperature Probe, data-collection interface, and calculator. (If you are using an EasyTemp, you do not need a data-collection interface.)
2. Set up EasyData for data collection.
 - a. Start the EasyData application, if it is not already running.
 - b. Select **(File)** from the Main screen, and then select **New** to reset the application.
 - c. Select **(Setup)** from the Main screen, then select **Time Graph...**
 - d. Select **(Edit)** on the Time Graph Settings screen.
 - e. Enter **10** as the time between samples in seconds.
 - f. Select **(Next)**.
 - g. Enter **120** as the number of samples and select **(Next)**.
 - h. Select **(OK)** to return to the Main screen
3. Determine room temperature. To do this, hold the probe in the air with nothing touching the probe tip. Observe the temperature reading on the calculator. When it is stable, record the value in your data table as the room temperature.
4. Push the Temperature Probe through the hole in the cap so that the end of the probe will be submerged in the water when the cap is on the canister. Do not let the end of the probe rest against the bottom of the canister.
5. Obtain some water at about 55°C. You should be able to get water this hot from a hot water faucet. If necessary, heat water to this temperature.
6. Carefully fill the canister about three-fourths full with the hot water. Place the cap containing the probe onto the canister and press until it is sealed with a click.
7. Wait about 10 seconds for the temperature probe to reach the temperature of the water. Collect your cooling data:
 - a. Select **(Start)** from the Main screen.
 - b. Collect data for 20 minutes, or until the temperature of the water is within 5° of room temperature. To stop data collection before the full 20 minutes have elapsed, select **(Stop)**.
8. Inspect your graph.
 - a. Sketch or print your graph.
 - b. Select **(Main)** to return to the Main screen.

DATA TABLE

Room Temperature (°C)	
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A	
B	
k	

ANALYSIS

- Since the model for Newton's law of cooling uses the difference between the sample temperature and room temperature, you must subtract the room temperature from the measured temperature before comparing data to the model.

- Select **Quit**, and then **OK** to quit EasyData.
- Press **2nd** [L2] **-** (room temperature) **STO>** L2 where (room temperature) is the numerical value you determined in Step 3 of the Procedure. This step replaces the measured water temperatures with the temperature above room temperature.
- Restart EasyData.

Fit the exponential function $y = A e^{-B \cdot x}$ to your temperature difference vs. time data.

- Select **Anlyz**, and then select **Exponential Fit** to fit an exponential function to your time data.
 - Record the fit parameters A and B in your data table.
 - Select **OK** to see a graph of your data with the fitted function.
- Newton's law of cooling was given above as

$$T_{diff} = T_0 e^{-kt}$$

Since you subtracted room temperature from the measured water temperatures, your graph shows the difference T_{diff} directly. The calculator fits the function $y = A e^{-B \cdot x}$ to your data. Match the variables x , y , A , B , and C in the fitted equation to terms T , T_{room} , k , and t in the expression of Newton's Cooling Law. Enter your value for k in the Data Table. What are the units of A and B ?

- When $t = 0$, what is the value of e^{-kt} ?
- When t is very large, what is the value of temperature difference? What is the temperature of the water at this time?
- What could you do to your experimental apparatus to decrease the value of k in another run? What quantity does k measure?
- Use your equation to calculate the temperature after 800 seconds. Compare your calculated value with the actual data value.
- Use your equation to predict the time it takes the water to reach a temperature 1°C above room temperature.

8. If the starting temperature difference is cut in half, does it take half as long to get to 1°C above room temperature?

EXTENSIONS

1. Take data for a longer period of time so that the water cools to nearly room temperature. This may take more than 30 minutes. Does the exponential model still fit the data?
2. A coffee drinker is faced with the following dilemma. She is not going to drink her hot coffee with cream for ten minutes, but wants it to still be as hot as possible. Is it better to immediately add the room-temperature cream, stir the coffee, and let it sit for ten minutes, or is it better to let the coffee sit for ten minutes and *then* add and stir in the cream? Which results in a higher temperature after ten minutes? Use your Temperature Probe to examine this dilemma. Explain your results in terms of the assumptions Newton made about cooling.
3. Use the Temperature Probe to experiment with coffee cups made of different material. Does a drink cool faster in a ceramic cup than in a Styrofoam cup? What variables must you hold constant in order to guarantee that the difference in the data is due to the cup? What part of the exponential equation is related to the cup?
4. The mathematical model for the cooling of a liquid can also be used to explain other phenomena in nature. For example, radioactivity and RC circuits behave in a similar fashion. Find other phenomena that are modeled by exponential functions. If possible make a measurement of the phenomenon in your physics lab.