Applications in Mathematical Reasoning

Who is the audience?

The primary audience for Applications in Mathematical Reasoning (AMR) are seniors in high school who are taking the course after completing Algebra II (the second year of algebra) and might prefer an alternative to pre-calculus. Approximately 25% percent of Washington State high school students take pre-calculus, although 77% of the students go on to some form of post-secondary education. It is imperative that we provide options for those students, because if we allow their mathematical skills to atrophy they will not be prepared for college or for most vocational training that leads to living-wage jobs.

For students who begin Algebra I in 8th grade, this course could be considered a math elective after pre-calculus. In addition, the level of rigor would be sufficient to engage students who have completed a year of calculus and who wish to improve their problem-solving abilities and reasoning skills.

Why is the course offered?

Pre-calculus and calculus are not for every student. The subjects are important to be sure for students wishing to pursue mathematics, science, engineering or related fields. But the type of abstraction that students encounter in that coursework is not appealing to everyone. Many important reasoning skills can be learned in the context of other mathematical applications. We can cast a wider net and keep more students engaged with mathematics by also offering coursework that highlights some of the beautiful and incredibly important applications in which mathematics plays a role, including game theory (with applications to everything from economics to military strategy to card games), social choice (including the mathematics of elections with many candidates), and probability and statistics (possibly the most important topics in mathematics for an educated citizen in a democracy).

We strongly believe that offering this course will help many more students see the beauty in mathematics, and it will better prepare them for what comes after high school: colleges, careers and their own lives. Many of the mathematical modules require discussion of social consequences which hopefully prepare students for many of the social issues they will face after high school and college.

What is the purpose of the course?

Our main goal in developing this course is to increase the enrollment of high school seniors in mathematics courses. The course content aligns with the College Readiness Mathematics Standards so that students will be prepared for either college-level math or technical training after graduation. We also hope to rekindle for students the joy of learning and the appreciation for the power of mathematics.

The main goal of students taking this course will be to develop their quantitative literacy – an important skill for citizenship, for the workplace, and for all postsecondary education. They will also expand and solidify their working knowledge of both algebra and geometry, thus preparing them for coursework in statistics, computer science, mathematics, technical fields, and the natural and social sciences.

AMR is a rigorous, applied and relevant course that goes beyond the scope of Algebra II and introduces mathematical topics that have been developed since 1945. The level of mathematics is engaging and accessible to students who have passed Algebra II. The content is ordered to reinforce algebraic skills so that students will be successful in college placement tests.

AMR is not intended to be a remedial course or a WASL preparation course. The level of critical thinking required of students dictates a higher level of quantitative literacy. In addition, the course is

not meant to be a review of algebra or an introduction to pre-calculus. (Topics from algebra, trigonometry and geometry are included in the course, but are used within new or extended applications.)

How is the course taught?

Course pedagogy and process are critical to the success of AMR. Professional development and support for instructors is required before and during the first year of teaching the course.

Each module begins with a challenging, interest-catching application. The instructors are encouraged to keep students actively engaged rather than using a lecture-type format. Students work in small groups after developing a classroom set of group norms. Literacy strategies and other study skills are encouraged throughout the course. Problem solving skills with authentic (real-world) activities are a daily occurrence. Class presentations require students to use good communication skills. Many of the activities are inquiry-based which help to develop necessary reasoning skills for students.

The resources for the class include the textbook, *For All Practical Purposes*, by CoMAP, a computer lab, graphing calculators and motion and temperature sensors. Many of the activities require additional supplies. Instructors receive a Teacher Resource Manual developed by Project TIME that includes a CD with the various spreadsheets needed for the class as well as a DVD with video clips that are used to introduce various modules. For each module, the manual describes the approximate length of the unit, daily lesson plans with the activities and their answer keys.

What is the content?

The content includes four main areas of mathematics. They are as follows.

- **Modeling with discrete structures** – This will be studied in contexts such as social choice and decision-making (i.e. game theory, fair division, and voting) and management science which is also called operations research (which includes graph theory, scheduling, and linear programming).
- **Quantifying uncertainty** –Students will investigate important topics in probability and statistics.
- **Quantifying shape** – This includes right-triangle trigonometry and two- and three-dimensional geometry.
- **Modeling with continuous functions** – Students will learn about linear, power, and exponential models and their various applications (including finance and the management of natural resources). Logistic and direct and inverse variation are optional units.

Specifically, the topics have been arranged in the following order so that certain topics occur at relevant times on the school calendar (such as voting just before November, investigating loans and the FAFSA form at the end of January when students are preparing for college). Each topic develops one or more of the Washington State College Readiness Math Standards (CRMS). The specific CRMS standards are listed under each module. For more information on the CRMS please see http://www.transitionmathproject.org/standards/index.asp

- Graph Theory: the abstraction of vertices and edges are utilized to represent locations and streets or distances with the goal of minimizing cost or time.
- Planning and Bin Packing: providing services in a timely and efficient manner requires planning. The goal is to minimize effort or time to get a set of tasks accomplished.
- Voting Theory: making social choices leads to the idea of how to arrive at a decision given differing opinions.

- Linear Programming: decision-making for businesses requires evaluating what resources are needed and how to allocate them.
- Statistics: understanding how to evaluate all of the data that is introduced daily requires knowing what data is relevant and how it can be used to make decisions.
- Probability: measuring uncertainty is the main goal.
- Fair Division: equitable division of assets and techniques to use are introduced.
- Finance: learning about loans, investments and credit cards are part of this module.
- Geometry: activities reinforce the idea of two and three dimensional measurement while incorporating algebraic techniques.
- Modeling with Continuous Functions: linear, quadratic, and exponential models are introduced. As time permits quartic, periodic and logistic models are included.
- Game Theory: the analysis of conflict, where two or more decision makers have different goals, is introduced.

Additionally, throughout the class, students are pushed to hone the attributes that are likely to make them successful in future coursework or employment:

- **Demonstrating intellectual engagement** - by actively exploring new ideas, posing questions about their meaning, significance and implications; taking risks and being challenged as part of the learning process; and contributing to group problem-solving activities.
- **Taking responsibility for learning** - evidenced by examining and learning form errors; and taking advantage of available resources.
- **Persevering** - through working on problems that require time and thought, particularly problems that cannot be solved by mimicking a previously seen example; recognizing when an approach is unproductive and making logical modifications; and successfully completing tasks that require organizing and implementing multiple steps, concepts or techniques.