LECTURE / STUDY NOTES

MATH 97
Green River Community College
Instructor: Kinholt
This set of lecture / study notes was created for students in MATH 97 – Intermediate Algebra at Green River Community College.

The notes are intended as a supplement to the textbook *Intermediate Algebra: A Just-in-Time Approach, Edition 3*, by Alice Kaseberg. Many of the examples, graphs, and figures can be found in this textbook.

You should use the notes in two ways:

1) Read each section of the text BEFORE it is presented in class. As you do, use the notes to focus your reading. Complete at least some of the problems that are presented in the notes BEFORE class.

2) Your instructor will use the notes during lectures. The information and problems presented in the notes will be used to highlight material from the textbook. During each lecture you will be able to check your answers to the lecture note problems that you have completed ahead of time and finish any remaining problems.

After you are done reading a section and working through some of the problems in the notes, attempt the homework problems that are assigned for that section. It is best if you attempt these homework problems BEFORE each class section. This will allow you to focus your attention during class on any problems you have encountered and to ask specific questions.

While the notes do not cover all of the material that you are responsible for, they will help you focus on the most important concepts in each section. While the notes should help you study for tests, they cannot be used during tests.

If you have any questions or comments about the Lecture/Study Notes, please contact Steve Kinholt at Green River CC: skinholt@greenriver.edu or (253) 833-9111 x4354
1. Define **function**: 

2. For people filing a single return, federal income tax is a function of adjusted gross income because for each value of adjusted gross income there is a specific tax to be paid. By contrast, the price of a house is not a function of the lot size on which the house sits because houses on same-sized lots can sell for many different prices.

   a) Describe an everyday situation that is a function.

   b) Describe an everyday situation that is not a function.

3. Federal income tax is a function of adjusted gross income. Is adjusted gross income the **input** or the **output**? (Remember the output depends on the input and the output is a function of the input.)

4. Define the following and tell which is the **input** and which is the **output**.
   
   **Domain:**

   **Range:**

5. See Example 1, pg. 84:
   a) Is the cost \( y = 3 + 2(x – 1) \), a function of the number of copies ordered? Why or why not?

   b) Why do we write \( 2(x – 1) \) and not \( 2x \)?

   c) What are the domain and range?

6. Determine whether the set of ordered pairs is a function. Give the domain and range.
   a) \( \{(1,2),(3,4),(5,5)\} \)   
   b) \( \{(4,1),(5,1),(6,1)\} \)   
   c) \( \{(1,1),(2,1),(1,4)\} \)
7. Explain how the **vertical line test** is used to determine whether a graph is a function? Draw a graph in which y is a function of x. Draw a graph in which y is not a function of x.

8. Which of the following represent functions?

![Graphs](image)

9. Functions can also be visualized as a “machine” that takes each input, performs a given operation on it using a “rule”, and produces one output based upon that rule. What is the “rule” for each function machine below?

![Function Machines](image)

10. Does the notation f(x) mean *f times x* or does it describe the *value of the function at x*? How is the notation f(x) read?

11. Function notation, f(x), has the advantage of allowing us to simultaneously name the __________ and the __________.

12. Evaluate the function f(x) = 3 + 2(x – 1) for the following inputs.
   a) f(1)  
   b) f(4)  
   c) f(n)

13. Find the following functions on your calculator:
   a) x²  
   b) √x  
   c) |x|  
   d) Function to convert .25 to a fraction  

   (look under CATALOG $\rightarrow$ [2nd]-[0])

   (look under MATH)
1. A linear equation in one variable can be written as \( ax + b = 0 \) where \( a, b \) are Real \( a \neq 0 \) 
A linear equation in two variables can be written as \( ax + by = c \) where \( a, b, c \) are Real \( a, b \neq 0 \) 
Is the equation linear? If so, one or two variables? Name the variables.

a) \( x = 4 \)  
   b) \( 2x - 3 = 9 \)  
   c) \( C = 2\pi r \)  
   d) \( x^2 = 4 \)

e) \( 3n - 2 = 11n + 14 \)  
   f) \( y = 2x + 3 \)

g) \( 4x^2 - 2x(2x + 1) = 10 \)

2. Define linear function:

3. Linear functions can be written in the form \( y = mx + b \) or \( f(x) = mx + b \) 
Is the function linear? If so, name \( m \) and \( b \).

a) \( f(x) = 10 - 0.10x \)  
   b) \( f(x) = 3 + 2(x - 1) \)  
   c) \( f(x) = 4 - x^2 \)

4. The table and graph below show data for a chocolate bar fundraiser. There is a required initial investment of $15 and the bars sell for $0.75.

<table>
<thead>
<tr>
<th>Bars Sold, ( x )</th>
<th>Total Profit, ( y = P(x) ) (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-15.00</td>
</tr>
<tr>
<td>5</td>
<td>-11.25</td>
</tr>
<tr>
<td>10</td>
<td>-7.50</td>
</tr>
<tr>
<td>15</td>
<td>-3.75</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
</tr>
</tbody>
</table>

a) Name the horizontal or \( x \)-intercept. What is the meaning of this intercept for this example?

b) Name the vertical or \( y \)-intercept. What is the meaning of this intercept for this example?

c) Slope is a rate of change: \( \frac{\Delta y}{\Delta x} \) Calculate the slope. What is the meaning for this example?
5. Make a table and then graph the following: $3x + 2y = 6$.

a) What is the x-intercept?  
b) What is the y-intercept?  
c) What is the slope?

6. Find the slope of the line that passes through the given pairs of points:
   a) $(-3, -5)$ and $(5, -1)$  
   b) $(-1\frac{1}{3}, 3\frac{2}{3})$ and $(1\frac{2}{3}, 2\frac{1}{3})$  
   c) $(6, 5)$ and $(6, -4)$

7. When a graph rises from left to right it represents an **increasing function**.  
   When a graph falls from left to right it represents a **decreasing function**.  
   In the space below, sketch: a) an increasing linear function; b) a decreasing linear function;  
   and c) an increasing non-linear function.

8. On the graph, sketch and label the following two lines:
   a) slope $\frac{2}{3}$, y intercept $-2$  
   b) slope 2, y intercept $-2$

9. For the following data, find:
   a) the slope and the units on the slope  
   b) the x-intercept and its meaning  
   c) the y-intercept and its meaning
1. Write the general form of the *Slope-Intercept Equation*.

2. Name the slope and y-intercept for: Total cost of a party at the pool given \( y = 3.00x + 10 \)

While your textbook also discusses the *point-slope form* for linear equations, the *slope-intercept form* can be used for all problems. This method only requires you to understand \( y = mx + b \).

For example, find the equation of the line in *slope-intercept form* given the points (2, 3) and (4,7).

   Step 1: Find the slope of the line.
   \( m = 2 \) (do you see why?)

   Step 2: Rewrite the equation with the slope.
   \( y = 2x + b \)

   Step 3: Select one of the ordered pairs and substitute in its x and y value in the equation \( y = 2x + b \). Say you selected the point (2,3).
   \( 3 = 2(2) + b \)

   Step 4: Solve for the y intercept or b.
   \( b = -1 \) (do you see why?)

   Step 5: Rewrite the equation with the y intercept.
   \( y = 2x -1 \)

3. Find the equation of the line in *slope-intercept form* which passes through (2,3) and (4, 4).

4. For each of the following, name the *slope* and *vertical or y-intercept*:

   a) The total cost for a pizza party: \( y = 3.50x \)

   b) Circumference is a function of radius: \( C = 2\pi r \)

   c) Distance traveled is a function of time: \( d = rt \)

5. Suppose blue jeans cost $17.80 per pair plus a $250 order handling fee. What is the fixed cost? What is the variable cost per pair? What is the cost function to buy x pairs of jeans?
6. The following is an example of an arithmetic sequence: \{9, 11, 13, 15, 17, \ldots\}

   a) Define an **arithmetic sequence**.

   b) Make a table of the arithmetic sequence.

   c) Using the table, calculate the slope.

   d) Look backwards through the table to determine the y-intercept (where x=0).

   e) Fit an equation to the data.

7. Define **linear regression**.

8. Using your graphing calculator, try and follow the steps outlined on the bottom of page 116 or on your calculator handout:

   a) Fit a linear equation to the following data.

<table>
<thead>
<tr>
<th>Cones</th>
<th>Value in dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>0</td>
<td>15.00</td>
</tr>
<tr>
<td>1</td>
<td>13.50</td>
</tr>
<tr>
<td>2</td>
<td>12.00</td>
</tr>
<tr>
<td>3</td>
<td>10.50</td>
</tr>
</tbody>
</table>

   b) What is the **coefficient of correlation** for this data?

9. Sketch what data points might look like for the following coefficients of correlation:

   a) \( r = +1 \)  
   b) \( r = -0.85 \)

   c) \( r = -1 \)  
   d) \( r = 0.12 \)
1. Horizontal and vertical lines also have slopes. Calculate the slope for each of the lines below by using the points that are provided.

   a) Slope of horizontal line (y = 4) _______
   
   b) Slope of vertical line (x = -6) _______

2. Graph the following lines and write their equations:

   a) Goes through (3,2) with slope of 0
      Equation ___________________
   
   b) Goes through (-2, 6) with undefined slope
      Equation ___________________
   
   c) Goes through the origin and has slope of 0
      Equation ___________________

3. The cost of renting a car is often based upon a base charge for insurance plus so much per mile driven. Suppose a compact car cost $0.10/mile plus a $30 base charge. A mid-sized car also cost $0.10/mile, but the base charge is $50.

   a) Write a cost function for the compact car. Calculate the cost for a 200 mile trip.

   b) Write a cost function for the mid-sized car. Calculate the cost for a 200 mile trip.

   c) What is the slope of each line? What is the meaning of the slope?

   d) What if the base cost for each was the same, but the cost per mile was different. How would the graph change?
4. How do the slopes of parallel lines compare?

5. **Perpendicular lines** intersect at a right or $90^\circ$ angle. This can happen in two ways.
   
a) If one line has a slope of 0 and another has a slope that is undefined, the lines are perpendicular.
   
b) If one slope has a slope of $\frac{a}{b}$ and another line has a slope which is the negative reciprocal of the slope of the first line $\frac{-b}{a}$, the lines are perpendicular.

Prove that the figure below is a rectangle by checking the slopes of the sides. Opposite sides should be ___________________ and adjacent sides should be ________________.

Slope AB ______
Slope BC ______
Slope CD ______
Slope DA ______

6. Find the equation of a line that is parallel to the line with equation $3x - 2y = 6$ and passes through the origin.

7. Find the equation of a line that is perpendicular to the line with equation $2x + 5y = 15$ and passing through $(1,4)$. 
1. Make a table for the following and then graph.
   Input: Number of movies you rent in a month.
   Output: Monthly payment of $15 for unlimited rentals.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

2. Note that no matter what the input, the output is always the same. We call this a **constant function**. What is the slope of a constant function? ______ What is the y-intercept of this function? ______ What is the domain? _____________ What is the range? _____________

3. Think of some real-life situations that are examples of constant functions.

4. Make a table for the following and then graph.
   Input: The amount of money you put into a stamp dispensing vending machine.
   Output: The value of the stamps dispensed.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

5. Note that no matter what the input, the output is always identical to the input. We call this an **identify function**. What is the slope of an identify function? ______ Does it pass through the origin? ______ What is the domain? _____________ What is the range? _______________ What is the equation of the identity function? _______________
6. Think of some real-life situations that are examples of identity functions.

7. Complete the tables below and then plot the two graphs on the same grid.

| x  | x+3   | |x + 3| |
|----|-------|---|------|
| -6 |       |   |      |
| -5 |       |   |      |
| -4 |       |   |      |
| -3 |       |   |      |
| -2 |       |   |      |
| -1 |       |   |      |
| 0  |       |   |      |
| 1  |       |   |      |

a) Describe how the two graphs compare.

b) The second graph is called an **Absolute Value** function. What are the coordinates of the “point” in this graph? ________ Why is it located here?

8. Suppose a birthday party costs $50 for up to 10 children, but then $4.50 for each additional child? This is called a **dot graph**. Sketch it in the space below. Would you connect the dots or points on this graph? Why or why not?

9. A telephone call costs $1.50 for the first minute and $0.10/minute for each additional minute. The graph for this function is called a **step graph**? Why are points connected with a line? Why are some circles solid and others open? What is the meaning of the two slashes on the graph?
1. Calculate the **first differences** and the **second differences** for each of the sequences below. Tell whether the sequence is linear, quadratic, or neither.

   a)  $-1, 2, 5, 8, 11, \ldots$
   b)  $24, 21, 16, 9, 0, \ldots$

   c)  $-1, 3, 13, 29, 51, \ldots$
   d)  $16, 15, 8, -11, -48, \ldots$

2. **Quadratic functions** are functions of degree two that may be written in the form: 
   $f(x) = ax^2 + bx + c$ where $a$, $b$, and $c$ are Real #’s and $x \neq 0$. The input variable is $x$. For each of the following, identify $a$, $b$, and $c$, as well as the input variable.

   a) $y = x(x + 1)$ 
      $a = \quad b = \quad c = \quad$ input var = 

   b) surface of sphere $f(r) = 4 \pi r^2$ 
      $a = \quad b = \quad c = \quad$ input var = 

   c) Triangular #s: $y = \frac{n^2 + 3n + 2}{2}$ 
      $a = \quad b = \quad c = \quad$ input var = 

   d) Height of cable $h(x) = \frac{4}{125} x^2$ 
      $a = \quad b = \quad c = \quad$ input var = 

3. Simplify to determine if these equations are quadratic. If they are, identify $a$, $b$, and $c$.

   a) $x - 4 = x(x+4)$

   b) $1 + x = x - 2(x+1)$

4. The graph of a linear function is a line and it can be graphed with ____ points. The graph of a quadratic function is a curve called a **parabola**. How many points would be needed to graph it?
5. Complete the table and then graph the parabola.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x) = 4 - x^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

a) Using both the table and the graph, locate the vertex.

b) Draw the line of symmetry. What is the equation of the line of symmetry? ______________

c) Solve these equations using both the table and the graph:

\[ 4 - x^2 = 3 \]
\[ 4 - x^2 = 0 \]
\[ 4 - x^2 = 4 \]

d) What are the x-intercepts? ________________ What is the y-intercept? ________________

e) What is the domain? _____________________ What is the range? ____________________

6. Make a table and graph for \( f(x) = x^2 + 5x + 6 \) and then try to answer some of the same questions as in #5 above.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x) = x^2 + 5x + 6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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</tbody>
</table>
1. A **parameter** is a letter representing a number that changes the orientation and position of the graph. What are the parameter in a linear function: \( y = mx + b \)?

2. Build a table for the sequence: 8, 15, 22, 29, 36, . . .

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
<th>1(^{st}) Dif</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   a) The first difference is the slope of the function. \( m = _____ \)

   b) \( f(0) \) is the vertical intercept. \( b = _____ \)

   c) Write the equation that generated this sequence: _______________________

3. Build a table for the sequence: 6, 21, 40, 63, 90, . . .

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
<th>1(^{st}) Dif</th>
<th>2(^{nd}) Dif</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
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<td>5</td>
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</tbody>
</table>

   To find the parameters \( a, b, \) and \( c \) for this quadratic function:

   a) You can find \( a \) by taking \( \frac{1}{2} \) of the second difference. \( a = _____ \)

   b) \( c = f(0) \). This is also the y-intercept. \( c = _____ \)

   c) Finally, to find \( b \), you substitute \( a, c, \) and a point into the general form for the quadratic \( f(x) = ax^2 + bx + c \) and then solve for \( b \). Write the equation: ________________________
4. To check your answer for problem #3, enter your equation into Y₁. Then under [TBLSET] let TblStart = 1 and eTbl = 1. Use [TABLE] to view the table and compare it to problem #3.

5. Similar to what you did for linear equations, you can also use your calculator to generate a quadratic function using the quadratic regression feature. Follow these steps:

Use [STAT]-[EDIT] to enter the data into lists L1 and L2
Under [STAT]-[CALC] choose 5:QuadReg
Don’t forget to press [ENTER] to run the regression.
Record the values for a, b, c, and r² and compare to your results in problem #3.

6. In quadratic regressions, r² has a similar meaning to the r value in linear regressions. Describe what r² means.

7. A discount store sells a 6-inch deep Duranco planter for $2.69, and 8-inch planter for $3.97, a 10-inch planter for $5.99, and a 12-inch planter for $8.97.

a) Use your calculator’s [STAT] feature to place the size and cost data into L1 and L2.

b) Graph the data. Do you think it fits a line or a quadratic (parabola) best?

c) Use linear regression to fit an equation. Equation ___________________ r = _____

d) Use quadratic regression to fit an equation Equation ___________________ r = _____

e) Graph both the linear and quadratic function. Note, you text has a hint on page 219 for how you can retrieve each of the equations using [VARS].

f) Using the quadratic function, find the cost for 14- and 16-inch pots.
1. Write examples of monomial, binomials, and trinomials.

2. Simplify the following. Add or subtract as indicated. Multiply where necessary.

   a) \((2x^2 + 3x - 5) - (6x^2 - 4x + 2)\)

   b) \(x(x^2 + 6x + 9) + 3(x^2 + 6x + 9)\)

   c) \((a^3 + a^2b + ab^2) - (a^2b + ab^2 + b^3)\)

3. Multiply the following using the tables provided.

   a) \((x - 6)(x + 3)\)

   b) \((2x - 3)(x + 4)\)

   c) \((2x + 1)(2x - 1)\)
4. Factor each of the following using the tables provided.

   a) \( x^2 + 4x - 12 \)

   b) \( 6x^2 + 7x - 10 \)

   c) \( 15x^2 - x - 6 \)

5. Fill in the blanks.

   a) Two or more numbers or expressions being multiplied are called______________.

   b) A one-term polynomial _______________ two-term _____________________
       three-term _______________ many-term ________________________

   c) The result of multiplying a number by itself ________________

   d) The result of multiplying three identical factors ________________

6. Factor these binomials:

   a) \( 3x^2 - 48 \)

   b) \( 28 - 63x^2 \)

   c) \( 18x^2 - 8 \)
1. If you multiply a binomial by itself, the result is a **perfect square trinomial**. Multiply the following:
   
   a) \((2x + 1)(2x + 1)\)
   
   b) \((x - 7)(x - 7)\)
   
   c) \((3x - 1)^2\)

2. Which of the following are perfect square trinomials?
   
   a) \(x^2 + 10x + 25\)
   
   b) \(x^2 + 6x + 9\)
   
   c) \(x^2 - 9\)

3. Use your calculator to graph the above three quadratics on the same grid. What do you notice about the two that are perfect square trinomials compared to the one that is not?
4. Factor $x^2 - 1$

5. Using your calculator, graph $y = x^2$ and $y = x^2 - 1$ on the same grid.

   a) How do they compare?

   b) Look at the factors in problem #4. Now, examine the graph of this function and describe its relationship to the factors.

   c) What do you think the graph of $y = x^2 - 4$ would look like? How about $y = x^2 - 9$?

Note: you can skip the section on Special Products: Cubic Expressions, pages 239-241.
1. **The zero-product rule** states that if the product of two expressions is zero, then at least one of the two expressions must be zero. (If \( A \cdot B = 0 \) then either \( A = 0 \) or \( B = 0 \)). Given this rule, explore the solutions for each of the following:
   a) \( A \cdot (-5) = 0 \)
   b) \( (x + 2) (x - 7) = 0 \)

2. Solve by factoring: \( x^2 - x - 6 = 0 \)
   Check your solution.
   Then, use your calculator to graph the function and discuss how the solutions relate to the graph.

3. Solve by factoring: \( x^2 + 2x + 1 = 16 \)
   Check your solution
   Then, use your calculator to graph the function and discuss how the solutions relate to the graph.

4. Solve by factoring: \( 3x(x + 1) - 5 = 31 \)
   Check your solution.
   Then, use your calculator to graph the function and discuss how the solutions relate to the graph.
5. The relationship between the solutions to a quadratic equation and its graph is shown in the figure below.

Using the equation $y = a(x - x_1)(x - x_2)$ you can build the equation for a quadratic when you know the two solutions (intercepts) and a third point. Use these steps:
- Substitute the two solutions or x-intercepts into the equation for $x_1$ and $x_2$.
- Substitute the third point in for $(x, y)$
- Solve for $a$
- You may then want to put the equation in standard form. $f(x) = ax^2 + bx + c$

a) Find the equation of the parabola passing through $(-2, 0)$, $(5,0)$, and $(4,3)$. Rewrite the equation in standard form.

b) Use your calculator’s quadratic regression feature to check your equation.

6. Using the same procedure outlined in problem #5, find the equation of the parabola passing through the points: $(-2, 12)$, $(-4, 0)$, and $(2, 0)$. Rewrite the equation in standard form. Again, use your calculator’s quadratic regression feature to check your equation.
1. A square root of $x$ is the real number that, when multiplied by itself, produces $x$.
Solve the following:

a) $x^2 = 36$  

b) $x^2 = 81$  

c) $x^2 = -16$  

d) $x^2 = \frac{1}{9}$

2. The principal square root of a number $x$ is only the positive real number that, when multiplied by itself, produces $x$. Find the principal square root for each of the following:

a) $\sqrt{36}$  

b) $\sqrt{81}$  

c) $\sqrt{-16}$  

d) $\sqrt{\frac{1}{9}}$

3. When solving $\sqrt{x}$ you list only one answer, the principal square root. When solving $x^2 = k$, you should list two solutions, $+\sqrt{k}$ and $-\sqrt{k}$
Solve the following and tell whether the solution is rational or irrational:

a) $\sqrt{6.25}$  

b) $\sqrt{8}$  

c) $\sqrt{0.04}$  

d) $\sqrt{-4}$

4. Estimate the solutions to each of the following. Then use your calculator and round solutions to the nearest thousandths.

a) $\frac{2-\sqrt{32}}{2}$

b) $\frac{5-\sqrt{50}}{5}$

5. Simplify the radicals without a calculator:

a) $\frac{4+\sqrt{8}}{4}$

b) $\frac{\sqrt{48}}{\sqrt{3}}$

c) $\sqrt{0.0025}$

d) $\sqrt{\frac{121}{49}}$
6. Given a right triangle \( \triangle ABC \) the two shorter sides that are perpendicular, \( AB \) and \( BC \), are called _______ and the longer side \( AC \) is called the __________________.

7. Describe in words what the relationship is between the sides of a right triangle.

8. What is the name of the theorem that describes this relationship? ____________________________

9. Carpenters know that the diagonals of a rectangle or square are equal. They use this fact to check that foundation forms have been built correctly. If the dimensions for a rectangular addition to a house are 16 feet by 20 feet, how long should the diagonal be?

10. Which of the following sets of numbers could form the lengths of sides of a right triangle?

   a) 8, 9, 12

   b) 11, 60, 61

   c) \( \sqrt{5}, \sqrt{11}, 4 \)
1. Given a quadratic in the form of \( ax^2 + bx + c = d \) where the trinomial on the left is a perfect square trinomial, you can solve the equation by factoring the left-hand side and then taking the square root of both sides.

   a) Solve: \( x^2 - 6x + 9 = 25 \)  
   b) Solve: \( 25x^2 + 10x + 1 = 9 \)

2. Try solving the problems from question #1 by factoring. Reminder, to solve by factoring, you must first make one side of the equation zero. Why?

   a) Solve: \( x^2 - 6x + 9 = 25 \)  
   b) Solve: \( 25x^2 + 10x + 1 = 9 \)

3. Given a quadratic in the form of \( ax^2 + bx + c \) and \( b = 0 \), it can easily be solved by taking square roots.

   a) Solve: \( x^2 - 121 = 0 \)  
   b) Solve: \( 5x^2 - 45 = 0 \)

4. Try solving the problems from question #3 by factoring.

   a) Solve: \( x^2 - 121 = 0 \)  
   b) Solve: \( 5x^2 - 45 = 0 \)

5. Suppose that each side of an equilateral triangle is 5 inches. Find its height.
6. The area of a circle with respect to its height is given by the formula: \( A = \pi r^2 \)
   Solve this equation for the radius, \( r \).

7. On the moon, the approximate distance in miles seen to the horizon from a height \( h \), in feet,
   is \( d = \sqrt{\frac{3h}{8}} \). Solve this equation for \( h \).

   How high must an astronaut on the moon climb in order to see a distance of 5 miles?

8. In auto accident investigations, the speed \( r \) of a car in miles per hour is estimated by measuring
   the length of the skid marks. (Round to the nearest whole number.) For a wet concrete road and
   skid marks of length \( L \), measured in feet, the formula for speed is \( r = \sqrt{12L} \)
   
   a) Solve the equation for \( L \).

   b) If a car is traveling at 55 miles per hour, how long will the skid marks be?

   c) If a car made a 100-foot skid mark, how fast was it traveling?
1. The following are perfect square trinomials. Explain how you can tell by looking at a trinomial if it is a perfect square trinomial.

\[ x^2 + 8x + 16 \quad x^2 + 4x + 4 \quad x^2 - 2x + 1 \]
\[ x^2 + 18x + 81 \quad x^2 - 10x + 25 \quad x^2 + 2x + 1 \]

2. The process of finding the term to add to \( x^2 + bx \) in order to create a perfect square trinomial is called **completing the square**. Complete the square on each of the following:

   a) \( x^2 - 5x \)  
   b) \( x^2 + 22x \)

3. Use completing the square to solve the following equations:
   a) \( x^2 - 6x + 8 = 0 \)  
   b) \( x^2 - 12x = -11 \)

4. Use completing the square on the general form: \( ax^2 + bx + c = 0 \)
5. The formula that results from completing the square of the general form (problem #4) is called the ___________________________. It can be used to solve ANY quadratic equation !!!

Use the quadratic formula to solve the following: \( \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

a) \( x^2 - 6x + 8 = 0 \) 

b) \( 5x^2 + 3x = -3 \)

6. The quadratic formula can also be used to find the vertex of a parabola. Study the illustration below and then find the vertex of the problem below (round to three decimal places).

\( y = x^2 + 4x - 5 \)

7. Try your calculator’s [MINIMUM] feature to check your answers to question number 6.

8. Study the **Vertical Motion Equation** below and then answer the questions that follow.

In terms of time \( t \), the height \( h \) of an object in vertical motion is

\[ h = -\frac{1}{2} gt^2 + v_0t + h_0 \]

where \( g \) is the acceleration due to gravity (9.81 m/sec\(^2\) or 32.2 ft/sec\(^2\)), \( v_0 \) is the initial upward velocity (in m/sec or ft/sec), and \( h_0 \) is the initial height (in meters or feet).

An Olympic diver stands on a diving platform 32.8 feet (10 meters) above the water. She jumps from the platform with an initial upward velocity, \( v_0 \), of 6 ft/sec. When will she hit the water? After how many seconds will her height be 23 feet? What will be her maximum height?
Section 6.1 Roles of $a, b, c$ in Graphing Quad Func

Remember a quadratic function may be written $f(x) = ax^2 + bx + c$. We will explore the role of parameters $a$, $b$, and $c$.

1. Complete the table and then sketch the graphs for: $y = x^2$ and $y = -x^2$
   a) How does the – sign if front of the $x^2$ change the numbers in the table from those generated by $x^2$?
   b) How does the graph of $x^2$ compare to that of $-x^2$?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x^2$</th>
<th>$-x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Complete the table and then sketch the graphs for: $y = x^2$, $y = 2x^2$ and $\frac{1}{2}x^2$
   How does the magnitude of $a$ affect the graph?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x^2$</th>
<th>$2x^2$</th>
<th>$\frac{1}{2}x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. The coefficient on the $x^2$ term, the parameter $a$, controls the shape and steepness of the graph.
   If $a$ is larger than 1, the graph is ______________ than the graph of $y = x^2$.
   If $a$ is between zero and 1, the graph is ______________ than the graph of $y = x^2$.
   If $a$ is negative, the graph is ______________.
4. To explore the role of the \( b \) parameter in \( f(x) = ax^2 + bx + c \), use your calculator to graph each of the following. Sketch the results on the grid.

You will see that the parameter \( b \) contributes to a change in the position of the vertex of the parabola. However, \( a \) and \( c \) also change the position of the vertex. No simple generalization is possible.

\[
\begin{align*}
y_1 &= x^2 \\
y_2 &= x^2 + 1x \\
y_3 &= x^2 + 2x \\
y_4 &= x^2 + 3x \\
\end{align*}
\]

5. To explore the role of the \( c \) parameter in \( f(x) = ax^2 + bx + c \), use your calculator to graph each of the following. Sketch the results on the grid.

How does the parameter \( c \) affect the graph?

\[
\begin{align*}
y_1 &= x^2 \\
y_2 &= x^2 + 2 \\
y_3 &= x^2 - 1 \\
\end{align*}
\]
In solving $x^2 + 1 = 0$, we obtain $x^2 = -1$, or $x = \pm \sqrt{-1}$. Although square roots of negative numbers are not real numbers and cannot be graphed on the rectangular coordinate axes, these numbers still have properties and applications. We define the **Imaginary Unit** as follows:

$$i = \sqrt{-1} \quad \text{or} \quad i^2 = -1$$

where $i$ is the **imaginary unit**.

1. Change these numbers into the product of a real number and the imaginary unit.
   a) $\sqrt{-4}$
   b) $\sqrt{-12}$
   c) $\sqrt{-32}$
   d) $\sqrt{-54}$

When we combine the **real numbers** with the **imaginary unit**, we create the **complex number system** and write them in the form of $a + bi$ where $a$ is the real number part and $bi$ is the imaginary part.

2. Write the following as complex numbers:
   a) $\sqrt{36}$
   b) $15$
   c) $4i$
   d) $\sqrt{-32}$
   e) $\frac{3 + \sqrt{-18}}{6}$

If $a$ and $b$ are real numbers, expressions of the form $a + bi$ and $a - bi$ are **complex conjugates**.

3. Write the **conjugate** of the following complex numbers:
   a) $6 + 3i$
   b) $-7 - 2i$

4. Adding and subtracting complex numbers. Simplify the following:
   a) $4i + 6 + 2 + 7i$
   b) $(4i + 6) - (7i + 2)$
5. Multiplying complex numbers. Simplify the following:

a) \((3 + 4i)(3 - 4i)\) 

b) \((\sqrt{2} + i)(\sqrt{2} - i)\)

The square root portion of the quadratic formula, \(\sqrt{b^2 - 4ac}\), controls the nature of the solutions. The expression under the radical, \(b^2 - 4ac\), is called the **discriminant**.

If \(b^2 - 4ac\) is **positive** there are ______ real-number solutions. The graph **passes** through the **x-axis** ______.

If \(b^2 - 4ac\) is **zero** there is _____ real-number solution, a double root. The graph **touches the x-axis** ______.

If \(b^2 - 4ac\) is **negative** there are _____ real-number solutions. The graph **doesn’t touch the x-axis**.

6. Write the discriminant for each of the following. Then, use your calculator to graph each on the same grid. Sketch the graphs on the grid below and then discuss how the discriminant relates to the graphs.

   a) \(f(x) = x^2 + 2x + 1\)

   b) \(f(x) = x^2 + 2\)

   c) \(f(x) = x^2 - 2x\)

7. Use your calculator to graph \(f(x) = x^3 - 1\).

   a) How many real solutions are there? _______ How can you tell?

   b) How many complex solutions are there? _______

   c) \(x^3 - 1\) factors into: \((x - 1)(x^2 + x + 1)\). Use this factorization to find all solutions. 
   
   *Hint, you may need to use the quadratic formula.*

   - 30 -
1. Use your calculator to graph each of the following equations. Observe a pattern between the equation containing the square binomial and the horizontal shift of its graph. Sketch the graphs.

   a) \( y_1 = (x + 1)^2 \)
   b) \( y_2 = (x - 2)^2 \)
   c) \( y_3 = (x + 3)^2 \)

   Now, without your calculator, sketch the graph of:

   \( y = (x - 4)^2 \)

   If the quadratic equation can be written as \( y = (x - g)^2 \), the vertex is at \((g, 0)\). The graph of \( y = (x - g)^2 \) is shifted horizontally \( g \) units from the graph of \( y = x^2 \).

2. Use your calculator to graph each of the following equations. Observe a pattern of the vertical shift of its graph. Sketch the graphs.

   a) \( y_1 = x^2 \)
   b) \( y_2 = x^2 + 1 \)
   c) \( y_3 = x^2 - 3 \)

   Now, without your calculator, sketch the graph of:

   \( y = x^2 + 4 \)

   If the quadratic equation can be written as \( y = x^2 + j \), the vertex is at \((0, j)\). The parabola for \( y = x^2 + j \) is shifted vertically \(|j|\) units from the graph of \( y = x^2 \).
3. Sketch the graph of the following equation without your calculator. Then, use your calculator to check your answer.

   a) \( y = (x - 2)^2 - 3 \)

   b) What is the vertex of this parabola? _______

   c) Explain how you can determine the vertex by looking at the equation.

The vertex form of a quadratic equation is \( y = a(x - h)^2 + k \), where the vertex coordinates are \((h, k)\).

4. If you are given the vertex of a parabola and a single other point on the parabola, you can write the equation for the parabola using the following steps:

   1. Start with the vertex form, \( y = a(x - h)^2 + k \).
   2. Using the vertex \((h, k)\), substitute for \(h\) and \(k\) in \( y = a(x - h)^2 + k \).
   3. Using the ordered pair for another point on the graph, substitute for \(x\) and \(y\).
   4. Solve for \(a\).
   5. Substitute \(a\) and the vertex into the vertex form.

Use this process to find the equation of the suspension bridge cable shown below. Let point A be the origin.

5. Use the process of completing the square to rewrite this quadratic equation in vertex form.
   \( y = x^2 + 6x + 5 \)  Graph on your calculator to see if you have the correct vertex.
In this section we examine the **maximum** or **minimum** point on a parabola, located at its vertex. To help with this, examine the following summary of information about the vertex.

<table>
<thead>
<tr>
<th>Summary: Vertex of the Parabolic Graph of ( y = ax^2 + bx + c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Once symmetry is observed in an input-output table, the vertex can be located at the highest or lowest output.</td>
</tr>
<tr>
<td>• The vertex is the highest or lowest point on a parabolic graph (Figure 23).</td>
</tr>
<tr>
<td>• The vertex lies on the axis of symmetry of a parabolic graph (Figure 23).</td>
</tr>
<tr>
<td>• The ( x )-coordinate of the vertex is the midpoint between the ( x )-intercepts (solutions to ( f(x) = 0 )).</td>
</tr>
<tr>
<td>• The ( x )-coordinate of the vertex is the midpoint of any horizontal segment, ( y = d ), that intersects the parabola in two points (Figure 23).</td>
</tr>
<tr>
<td>• The vertex has the ordered pair ((x, y)), where ( x = -b/2a ) and ( y = f(-b/2a) ).</td>
</tr>
<tr>
<td>• The vertex is ((h, k)) when we have completed the square on a quadratic equation and changed it into vertex form, ( y = a(x - h)^2 + k )</td>
</tr>
</tbody>
</table>

**FIGURE 23**

1. One way to find the vertex is to first find the \( x \)-intercepts, if they exist.

   a) Factor \( f(x) = x^2 - 4x - 12 \)

   b) Check the **discriminant** to verify that there are \( x \)-intercepts.

   c) What are the \( x \)-intercepts?

   d) What point is halfway between the \( x \)-intercepts?

   e) What is the equation for the line of symmetry?

   f) Using your answer from \( d \), substitute into the equation to find the \( y \)-coordinate for the vertex.

   g) Write the vertex as an ordered pair.

   h) Is this point a maximum or a minimum? Explain.

   i) Use your calculator’s **[CALC]-[MINIMUM]** or **[MAXIMUM]** function to check your answer. You may have to adjust your **[WINDOW]**.

   j) Examine **[TABLE]** data to verify your answer.
2. The quadratic formula can help you locate the vertex of a parabola. The x-coordinate of the vertex is \( \frac{-b}{2a} \). Do you see how this relates to the quadratic formula? 

\[ \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

To find the y-coordinate, simply substitute the x-coordinate in the equation and solve for y.

\[ \left( \frac{-b}{2a}, f\left( \frac{-b}{2a} \right) \right) \]

A bridge support arch has the equation \( y = -\frac{2}{1125}x^2 + \frac{8}{15}x + 25 \), with \( x \) in feet. Find the clearance between the arch vertex and the water if the water is at the level of the x-axis.

3. When an object is launched, the height above ground level relative to the time in the air behaves according to the equation

\[ h = -0.5gt^2 + v_0t + h_0 \]

where \( g \) is acceleration due to gravity (32.2 ft/sec\(^2\)), \( v_0 \) is the initial velocity, and \( h_0 \) is the initial height.

A ball of fireworks is shot vertically from the ground with an initial velocity of 115 ft/sec. The fuse is set to go off at the maximum height. Find the time until the fireworks burst and the height at which they burst.
A **ratio** is the *quotient of two quantities*. Ratios compare like or unlike quantities. Slope is an example of a ratio. Pi (π) is another example. Two ratios are **equivalent ratios** if they simplify to the same number.

1. The ratio $\frac{22}{7}$ is sometimes used as an approximation for $\pi$. Using your calculator, compare this approximation and $\frac{2218}{706}$ with the approximation that is programmed into your calculator.

2. Simplify the following ratios:
   
   - a) $\frac{50m^2}{25m}$
   - b) $\frac{300 \text{ foot pounds}}{10 \text{ pounds}}$
   - c) $\frac{36 \text{ in.}}{1728 \text{ in.}^2}$
   - d) $\frac{1200 \text{kilowatt hours}}{24 \text{ hours}}$

**Unit analysis** is a method *for changing from one unit of measure to another or for changing from one rate to another*. A key idea of unit analysis is that you can multiply a ratio by 1 to change its units without changing the quantity that is represented. One (1) can be represented in different ways:

\[
\frac{12 \text{ inches}}{1 \text{ foot}} = 1 \quad \frac{360 \text{ day}}{1 \text{ year}} = 1 \quad \frac{1 \text{ kilogram}}{1000 \text{ grams}} = 1
\]

3. Use unit analysis for the following:
   
   a) 25 feet is how many meters? (1 ft = 12 in; 1 m ≈ 39.37 in)
   
   b) 100 cubic inches is how many cubic feet? (12 in = 1 ft.)

4. Simplify the following:
   
   - a) $\frac{1 \text{ foot}}{4 \text{ inches}}$
   - b) $\frac{12 \text{ min}}{\frac{1}{2} \text{ hour}}$
A rate is a comparison of a quantity of one unit to a quantity of another unit. Unit analysis can be used to change from one rate to another.

5. Use unit analysis to change the following:
   a) 40 miles per hour is how many feet per second?
      1 mile = 5280 feet
      1 minute = 60 seconds
      1 hour = 60 minutes

   b) Calculate your walking speed in feet per second and then change it to miles per hour.

   Two equal ratios form a proportion: \( \frac{a}{b} = \frac{c}{d} \), where \( b \neq 0 \) and \( d \neq 0 \)

6. Use proportions and their cross-products to find the missing value.
   a) \( \frac{4}{13} = \frac{x}{7} \)
   b) An access ramp is to have a 1 to 12 ratio.
      If it must rise 5ft, what must its length be?

   c) A small tree has been planted 10 feet from a streetlight. The streetlight is 20 feet tall. The light creates a tree shadow 16 feet long. What is the height of the tree?

   d) Find the missing sides:
Section 7.2 Proportions & Direct Variation (pages 362-370)

In this section, concentrate on pages 362-365. You are not responsible for the quadratic and joint variation sections.

Not all linear data represents proportional data. For this to happen, the ratio of the output to the input for each pair of data points must be the same.

1. Which of the following sets of data is proportional?
   
a) Tuition and fees are $185 for 2 credit hours and $345 for 4 credit hours.

   b) A bicyclist travels 15 miles in 1 hour and 45 miles in 3 hours.

2. Write the linear equations for the problems in exercise #1. Write in the form of $y = mx + b$.
   
a)

   b)

3. By examining the equations above, describe how you can tell if linear data is proportional.

   Direct variation occurs when the ratio of outputs to inputs for all data is constant. If the data is linear we say that it varies linearly.

   $$\frac{y_1}{x_1} = \frac{y_2}{x_2}$$

   If there is a constant $k$ such that $y = kx$, we say $y$ varies linearly as $x$. The ratio of outputs to inputs is $k$ for all ordered pairs, and the line connecting the data points passes through the origin.
4. Two cans of refried beans cost 98 cents. Fifteen cans of refried beans cost $7.35.
   a) Is this an example of direct variation? Explain.
   b) If “yes” to part a, write a sentence to describe this variation using the phrase “varies directly”.
   c) Write the linear equation.
   d) What is the slope and what does it mean?
   e) What is the y-intercept?
   f) What is the constant of variation for this problem? How does it compare to the slope?

5. Read the two problems below and study their graphs. Discuss which ones represent linear data and which represent proportional linear data.
   a) Repairing a 12-foot length of sidewalk takes 12 hours. The same contractor takes 17 hours to repair a 32-foot length of sidewalk.
   b) One 15-minute phone call costs $12.45. Another call to the same place at the same time of day costs $6.64 for 8 minutes.

There are two common ways of writing linear variation. The ratio form is \( \frac{y}{x} = k \). The function form is \( f(x) = kx \) or \( y = kx \). The constant ratio, \( k \), is called the constant of variation (or constant of proportionality).

6. The circumference of a circle varies directly and linearly with its diameter. Express this relationship in ratio form and function form. What is the constant of variation?
1. Suppose the current lottery jackpot is $24 million. You know that you have the winning numbers but there may be up to 24 other winners!

a) Complete the table below.

<table>
<thead>
<tr>
<th>Prize (dollars)</th>
<th>Lottery Jackpot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
</tr>
</tbody>
</table>

b) Sketch a graph of the data

c) How many winners would there have to be to reduce the winnings to $1?

d) Is this graph parabolic?

2. Using the information provided here, determine whether the data in example #1 represents direct or inverse variation. What is the constant of variation for this problem? 

For data \((x_1, y_1) \) and \((x_2, y_2)\):

Direct variation has a constant ratio \(y_1/x_1 = y_2/x_2\), or \(y/x = k\), the variation constant.

Inverse variation has a constant product \(x_1y_1 = x_2y_2\), or \(x \cdot y = k\), the variation constant.

3. Write each of the following as an equation with \(x\), \(y\), and \(k\). Write the value of \(k\).

a) The number of points given to each problem on a 100-point test varies inversely with the number of problems on the test.

\[ k = \ldots \]

b) The number of years the world resources of silver will last varies inversely with the number of metric tons used per year. The estimated world supply in 2001 was 430,000 metric tons.

\[ k = \ldots \]
4. Look at the following three tables and determine if the data represents **direct** or **inverse** variation. For each, identify if there is a constant ratio or a constant product. For each, write the constant of variation.

a) **Servings in a Gallon of Milk**

<table>
<thead>
<tr>
<th>Size of Glass (ounces)</th>
<th>Number of Glasses</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>21(\frac{1}{2})</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>12</td>
<td>10(\frac{1}{2})</td>
</tr>
</tbody>
</table>

Direct or Inverse \( k = \) _____

b) **Clothing Purchase**

<table>
<thead>
<tr>
<th>Number of Jeans</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$ 56</td>
</tr>
<tr>
<td>10</td>
<td>$280</td>
</tr>
</tbody>
</table>

Direct or Inverse \( k = \) _____

c) **Real Cost of Purchase**

<table>
<thead>
<tr>
<th>Times to Wear Blouse</th>
<th>Cost per Wearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$26.00</td>
</tr>
<tr>
<td>10</td>
<td>$ 5.20</td>
</tr>
</tbody>
</table>

Direct or Inverse \( k = \) _____

5. There is an **inverse relationship** between the weights needed to balance a lever (or seesaw) and the distance the weights are from the center or fulcrum. Assume that you are sitting 10 feet from the fulcrum of a seesaw and that you weigh 150 lbs. Where would your friend have to sit to balance the seesaw if she weighs 120 lbs? *(Draw a sketch!)*

Quantities are **inversely proportional** if their product is constant: \( x \cdot y = k \). For paired data, such as \((x_1, y_1)\) and \((x_2, y_2)\), being inversely proportional means that \(x_1 \cdot y_1 = x_2 \cdot y_2\). The constant product \( k \) is called the constant of variation (or constant of proportionality).

\[
\frac{y_1}{x_1} = \frac{y_2}{x_2} \quad \text{Direct variation} \quad \frac{y_1}{x_2} = \frac{y_2}{x_1} \quad \text{Indirect variation}
\]
Rational numbers are the set of numbers that may be written as the ratio of two integers \( \frac{a}{b} \), with \( b \neq 0 \). A rational expression is formed by the quotient of two polynomials.

1. When would the following rational expressions be undefined?

   a) \( \frac{x+3}{2-x} \) 
   b) \( \frac{a}{a-b} \) 
   c) \( \frac{x(x-3)}{x^2-3x-28} \)

   The factors \((x - y)\) and \((y - x)\) are additive inverses, or opposites. 
   Additive inverses add to zero.

   A rational expression \( \frac{x-y}{y-x} \), containing additive inverses in the numerator 
   and denominator, simplifies to \(-1\).

   A negative sign on a fraction may be placed in any of three positions—in the 
   numerator, in the denominator, or before the fraction. Thus, these fractions all 
   have the same value:

   \[
   \frac{-a}{b} = \frac{a}{-b} = \frac{-a}{b}, \quad b \neq 0
   \]

2. Some of the following simplify to 1, others simplify to \(-1\), and others do not simplify.

   a) \( \frac{x-4}{x-4} \) 
   b) \( \frac{-x-2}{x+2} \) 
   c) \( \frac{x+3}{3+x} \) 

   d) \( \frac{x-2}{2-x} \) 
   e) \( \frac{x+6}{x-6} \) 
   f) \( \frac{-x+5}{x-5} \)

3. Can the following be simplified as shown?

   a) \( \frac{4 \times 6}{6} \) 
   b) \( \frac{x \times 2}{2} \) 
   c) \( \frac{x^2 \times \sqrt{2}}{\sqrt{x} \times \sqrt{2}} \)

4. Simplify the following rational expressions and state any restrictions on the denominators.

   a) \( \frac{-49x^3y^5}{63x^2y^5} \) 
   b) \( \frac{8xy^2}{2xy} \) 
   c) \( \frac{2xy}{10x^3y^2} \)
5. Simplify the following by first factoring each numerator and each denominator. State any restrictions on the denominators.

a) \( \frac{xy}{2x + xy} \)  
   b) \( \frac{x^2 - 16}{x^2 + 3x - 4} \)  
   c) \( \frac{y^2}{6x^2 + y} \)

d) \( \frac{x^2 + x - 6}{2 - x} \)  
   e) \( \frac{x - 4}{12 - 3x} \)

6. Multiply or divide as indicated. Assume no zero denominators.

a) \( \frac{18}{cd} \div \frac{4d^2}{9c} \)  
   b) \( \frac{x + 2}{x^2 - 4x + 4} \cdot \frac{x^2 - 2x}{x + 2} \)

   c) \( \frac{x^2 + 3x}{x} \cdot \frac{x^2 - x - 6}{x^2 - 9} \)  
   d) \( \frac{y^2 - x^2}{x^2 + 2xy + y^2} \div \frac{x - y}{x + y} \)

7. Write each as a complex fraction and then simplify.

a) The quotient of \( \frac{1}{a} \) and \( \frac{1}{b} \)  
   b) The quotient of \( \frac{a}{b} \) and \( \frac{1}{b} \)

   c) \( \frac{x^2 - 4}{x^2} \div \frac{x^2}{x^2 + 5x + 6} \)  
   d) \( \frac{500 \text{ miles per hour}}{200 \text{ gallons per hour}} \)
1. Change the fractions to equivalent fractions with the indicated denominator. State any restrictions on the denominators.

   a) \( \frac{5}{9} = \frac{b}{54} \)
   
   b) \( \frac{4}{x} = \frac{x}{2x^3} \)

   c) \( \frac{b}{b + 5} = \frac{3b(b + 5)}{3b} \)
   
   d) \( \frac{x}{x + 2} = \frac{x^2 + 3x + 2}{x^2 + 3x + 2} \)

2. Add or subtract as indicated. State any restrictions on the denominator.

   a) \( \frac{2}{5} + \frac{x}{5} \)
   
   b) \( \frac{2}{3x} - \frac{5}{3x} \)

   c) \( \frac{-9}{x + 3} + \frac{x^2}{x + 3} \)
   
   d) \( \frac{2}{x^2 - 1} - \frac{x + 1}{x^2 - 1} \)

---

To find the least common denominator:

1. List the prime factors of each denominator.
2. Compare the lists of prime factors.
3. a. If the lists have no common factors, the LCD is the product of the denominators.
   
   b. If the lists have common factors, write each factor the highest number of times it appears in any one denominator. The LCD is the product of these factors.
3. What is the common denominator in each of the following? Rewrite each fraction with the common denominator and then add or subtract as indicated. Assume no zero denominators.

a) \( \frac{2}{a} - \frac{5}{2a} \)

b) \( \frac{b}{a^2} - \frac{c}{ab^2} \)

c) \( \frac{x}{x-3} + \frac{3}{x^2 - 6x + 9} \)

d) \( \frac{5}{x^2 + x} + \frac{x}{x^2 - 2x - 3} \)

4. Simplify the complex rational expressions to eliminate the fractions from the numerator and the denominator.

a) \( \frac{x}{3} - \frac{x}{3 + x} \)

b) \( h = \frac{V}{\pi d^2} \)

5. Working alone, Hose A can fill a swimming pool in 8 hours. Working alone, Hose B can fill that same swimming pool in 6 hours. How long would it take to fill the swimming pool with both hoses working together?
1. Solve the following equation. Then, graph each side of the equation as a separate equation to check your solution.

\[ \frac{3}{5}x - 8 = 13 \]

2. Solve for the indicated variable:

   a) \[ L = \frac{\pi r \theta}{180} \] for \( r \)

   \[ \frac{4 - 3x}{2} = \frac{3 - 5x}{3} \]

3. Solve and note any restrictions on the variables.

   a) \[ \frac{x - 7}{10} = \frac{-3}{x + 6} \]

   b) \[ \frac{1}{8} + \frac{1}{x} = \frac{1}{6} \]
4. Solve and note any restrictions on the variables.

a) \[ \frac{4}{x-1} = \frac{5-x}{x-1} + 2 \]

b) \[ \frac{1}{x} + \frac{x+1}{x+2} = \frac{6x-1}{5x} \]

5. An extraneous root is a solution found algebraically that does not satisfy the original equation. Check to see if the following results in an extraneous solution.

\[ \frac{x+1}{x-1} = \frac{1}{1-x} \]

6. The formula for calculating resistance for parallel resistors is: \( \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R} \)

Two resistors are in parallel with \( R_1 = 10,000 \) ohms and \( R_2 = 4000 \) ohms. Find the total resistance \( R \).
When an exponent is applied to a base, the result is called the power. \( \text{Base}^{\text{exponent}} = \text{Power} \)

1. Define what an exponent means? Also, discuss whether this definition applies to exponents that are positive integers? Zero? Negative integers? Rational integers?

2. Complete the following properties of exponents and then show a specific example:
   a) Multiply numbers with like bases: \( x^a \cdot x^b = \)
   b) Divide numbers with like bases: \( \frac{x^a}{x^b} = \)
   c) Exponent applied to a power: \( (x^a)^b = \)
   d) Exponent on the outside of parentheses applies to all parts of a product or quotient:
      \[ (x \cdot y)^a = \qquad \left( \frac{x}{y} \right)^a = \]

3. Simplify the expressions:
   a) \( x^3 \cdot x^3 \) \hspace{1cm} b) \( (ab)^3 \) \hspace{1cm} c) \( (4n)^2 \)

4. Simplify the expressions:
   a) \( (x^2)^4 \) \hspace{1cm} b) \( (2x^3)^5 \) \hspace{1cm} c) \( (3a^4b^2)^3 \)

5. Simplify the expressions:
   a) \( (x^a)^4 \) \hspace{1cm} b) \( \frac{xy^3}{x^3y} \) \hspace{1cm} c) \( \frac{x^k y^l}{x y^2} \)
6. Complete the following table by looking at the pattern and then discuss the meaning of a zero or negative integer exponent.

<table>
<thead>
<tr>
<th>TERM</th>
<th>2^5</th>
<th>2^4</th>
<th>2^3</th>
<th>2^2</th>
<th>2^1</th>
<th>2^0</th>
<th>2^-1</th>
<th>2^-2</th>
<th>2^-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>POWER</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Simplify each of the following:
   a) 5^0  
   b) (3.5)^0  
   c) 5^{-1}  
   d) \left(\frac{1}{2}\right)^{-1}  
   e) \left(\frac{2}{5}\right)^{-1}

   The **zero power of a nonzero base** is defined as 1:
   \[ b^0 = 1, \quad b \neq 0 \]

   The expression 0^0 is not defined.

   The **negative one power of a nonzero base** is defined as the reciprocal of the base:
   \[ b^{-1} = \frac{1}{b}, \quad b \neq 0 \]

   The expression 1/0 is not defined.

   The **negative one power of 1/b** is also the reciprocal:
   \[ \left(\frac{1}{b}\right)^{-1} = \frac{b}{1}, \text{ or } b \]

8. Simplify the expressions. Leave no negative or zero exponents.
   a) 3y^{-1}  
   b) \left(\frac{a}{b}\right)^{-1}  
   c) \left(\frac{a}{2c}\right)^{-3}  
   d) \frac{2x^3y^{-2}}{6x^{-1}y^2}  
   e) b^1b^n

9. Name the following as adding like terms, multiplying like bases, or neither. Then simplify.
   a) x^2 + x^{-2}  
   b) y^{-3} + y^{-3}  
   c) y^{-4}y^2
1. Define **scientific notation**. Describe its application.

To change scientific notation to regular decimal notation and the reverse, remember that numbers larger than 1 have a positive exponent on the 10 and small numbers between 0 and 1 have a negative exponent on the 10.

2. Change the following decimals to scientific notation.
   a) 100,000
eq 10^5
b) 0.000025

c) distance from sun to Earth in miles
eq 94,600,000
d) mass of a bacterium
\approx 0.000 000 000 0001

3. Change the following from scientific notation to decimal notation.
   a) 3.6\times10^2
   b) 3.6\times10^{-3}

   c) mass of a proton
   \approx 1.6726\times10^{-24} \text{g}
   d) current national debt
   \approx 7.376\times10^{12}

---

*Changing to scientific notation:* Look for options under [MODE] to change to scientific notation. In Sci mode, any number entered into the calculator will be changed automatically to scientific notation. The calculator displays for 25,000 and 0.000 025 are in Figure 2a. The calculator shows \( e \) to represent “times 10 raised to the exponent.”

*Changing to decimal notation:* When in Normal mode, a graphing calculator will change any number in scientific notation to decimal notation if the decimal number will fit on the display. Otherwise, the number will remain in scientific notation. Use 2nd [EE]—not [E^x], [10^x], or [\text{\underline{X}}]]—to enter a number in scientific notation. The EE signifies “enter exponent.” Graphing calculators permit entering the negative sign before the exponent. A calculator display in Normal mode for 9.46 \times 10^7 and 9.109 \times 10^{-31} is shown in Figure 2b.
4. Estimate the following by doing them mentally. Then, use the scientific notation feature of your calculator to check your estimate.

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Calculator</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( (5.0 \times 10^3)(2.5 \times 10^{-4}) )</td>
<td>( (5.0 \times 10^3)(2.5 \times 10^{-4}) )</td>
</tr>
<tr>
<td>b) ( (6.0 \times 10^{-3})(2.5 \times 10^{-4}) )</td>
<td>( (6.0 \times 10^{-3})(2.5 \times 10^{-4}) )</td>
</tr>
<tr>
<td>c) ( \frac{7.5 \times 10^{-2}}{2.5 \times 10^4} )</td>
<td>( \frac{7.5 \times 10^{-2}}{2.5 \times 10^4} )</td>
</tr>
<tr>
<td>d) ( \frac{2.4 \times 10^6}{0.3 \times 10^{-2}} )</td>
<td>( \frac{2.4 \times 10^6}{0.3 \times 10^{-2}} )</td>
</tr>
</tbody>
</table>

5. When writing numbers in scientific notation, you look for significant digits. Significant digits are all non-zero digits and certain zeros:
   - zeros between nonzero digits (as in 707)
   - zeros following a nonzero digit after a decimal place (0.020)
   - zero that are placeholders and marked with an overbar (as in \( 5\overline{00} \))

Write the following in standard notation:

a) \( 0.05\overline{0} \times 10^{-4} \)  b) \( 2.50 \times 10^3 \)

Write the following in scientific notation:

\( 25\overline{00} \)  \( 2090 \)

\( 0.01090 \)
1. Use your calculator and “guess and check” to solve the following problems:
   a) \( 2^n = 256 \)  
   b) \( 4^n = 256 \)  
   c) \( 16^n = 256 \)  
   d) \( 8^n = 256 \)

2. Now solve the problems in #1 above again, but rewrite each as factors of 2. Part c is done for you.
   a) \( 2^n = 256 \)
   b) \( 4^n = 256 \)
   c) \( 16^n = 256 \) \((2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \) 2 sets of 16 so \( n=2 \)
   d) \( 8^n = 256 \)

3. Now try to solve the following using a method like that above. The first one is done for you.
   a) \( 3^2 = (2 \cdot 2) \cdot 2 = 8 \)  
   b) \( 27^\frac{2}{3} \)
   c) \( 64^\frac{1}{3} \)  
   d) \( 25^\frac{1}{2} \)
   e) \( 64^\frac{1}{6} \)  
   f) \( 16^\frac{5}{4} \)
   g) \( 25^{1.5} \)  
   h) \( 0.09^{0.5} \)
4. Suppose that you were just informed that an ancestor of yours had deposited $1.00 in a savings account 200 years ago, it has continued to earn interest at a rate of about 7% per year, and the money is now yours! How much is in the account now?

5. If you borrow $200 from a payday lender at 1% per day, what will you owe in 30 days?

6. Assume you start with $1000 in an account.
   a) How much will you have in 1.5 years at 5%, with monthly compounding?
   
   
   b) How much will you have in $4\frac{3}{4}$ years at 7%, with quarterly compounding?
1. In the following, identify the **radical**, the **radicand**, and the **index**.

\[ n^{th} \ text{ root} = \sqrt[n]{x} \]

2. Use your calculators [MATH] – [ \( \sqrt[n]{ \ldots } \) ] function to estimate the following. *Note: enter the index before selecting the function.*

   a) \( \sqrt[8]{4096} \)  
   b) \( \sqrt[5]{40} \)

3. Simplify the following roots of negative numbers, if possible:

   a) \( \sqrt[3]{-8} \)  
   b) \( \sqrt[5]{-243} \)

   c) \( -\sqrt{16} \)  
   d) \( \sqrt{-16} \)

\[ \text{Let } x \text{ be a nonzero real number and let } n \text{ be a positive number. If } \sqrt[n]{x} \text{ is defined, the rational exponent } 1/n \text{ means} \]

\[ x^{1/n} = \sqrt[n]{x} \]

\[ \text{If } 1/n \text{ is a positive rational number, } 0^{1/n} = 0. \]

4. Write each rational exponent expression as a root. Write each root as a rational exponent expression.

   a) \( x^{\frac{1}{3}} \)  
   b) \( \sqrt[3]{x} \)  
   c) \( x^{0.25} \)  
   d) \( x^{0.2} \)

5. Simplify each of the following.

   a) \( (\sqrt[3]{-8})^5 \)  
   b) \( \sqrt[3]{27^2} \)  
   c) \( (\sqrt[3]{-27})^2 \)  
   d) \( 4\sqrt{-16} \)
Like Bases Property: \( x^a \cdot x^b = x^{a+b} \) \( \frac{x^a}{x^b} = x^{a-b} \)

6. Simply each, assuming that \( x > 0 \).

   a) \( \sqrt[3]{x^2} \cdot \sqrt[3]{x} \)  
   b) \( \frac{1}{a^2} \cdot a^3 \)  
   c) \( \frac{x^3}{x^2} \)  
   d) \( \frac{\sqrt[3]{x^4}}{\sqrt[3]{x^2}} \)

Power of a Power & \( n \)th Root of an \( m \)th Root Properties: \( (x^a)^b = x^{ab} \) \( \sqrt[ny]{x} = \sqrt[ny]{x} \)

7. Simplify, where \( a, b, \) and \( y \) are any real numbers.

   a) \( (a^2)^{\frac{1}{2}} \)  
   b) \( (b^3)^{\frac{3}{5}} \)  
   c) \( \sqrt[3]{x} \)  
   d) \( \sqrt[\sqrt{3}]{x} \)

---

Power of a Product, and Power of a Quotient, \( n \)th Root of a Product, and \( n \)th Root of a Quotient Properties

For positive bases, a rational exponent outside the parentheses applies to all parts of a product or quotient inside the parentheses:

\[
(x \cdot y)^a = x^a \cdot y^a \quad \text{Power of a product property}
\]

\[
\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} \quad \text{for } y \neq 0 \quad \text{Power of a quotient property}
\]

An index applies to all parts of a product or quotient in the radicand:

\[
\sqrt[n]{xy} = \sqrt[n]{x} \cdot \sqrt[n]{y} \quad x > 0, y > 0 \quad \text{Root of a product}
\]

\[
\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}} \quad x > 0, y > 0 \quad \text{Root of a quotient}
\]

---

8. Simplify the following:

   a) \( \sqrt{2} \sqrt{18} \)  
   b) \( \sqrt[3]{3\sqrt{64}} \)

   c) \( \sqrt[3]{9} \sqrt[3]{3} \)  
   d) \( \frac{81}{\sqrt{3}} \)  
   e) \( x^3 x^{\frac{1}{2}} \)

   f) \( \sqrt[3]{81} \)  
   g) \( \sqrt[3]{x^4 y^6 z^{10}} \)  
   h) \( \sqrt[3]{64 y^{12}} \)
Just as we can add and subtract like terms in polynomials, we can add and subtract similar radicals. **Similar radicals** have *identical indices and radicands*.

1. Add or subtract if possible.
   a) \(3\sqrt{2} - 4\sqrt{2} + 7\sqrt{2}\)  
   b) \(\sqrt{50} + \sqrt{32}\)  
   c) \(3\sqrt{x} - \sqrt{x}\)  
   d) \(3\sqrt{x} - 2\sqrt{x}\)  
   e) \(\sqrt{64x} - \sqrt{27x}\)  
   f) \(\sqrt{16x^4y} + x\sqrt{y}\)

2. Multiply and simplify.
   a) \((2 - \sqrt{3})(2 + \sqrt{3})\)
   b) \((5 - \sqrt{2})(5 - \sqrt{2})\)
   c) \((x - \sqrt{3})(x - \sqrt{3})\)
   d) \((2 - \sqrt{a})^2\)

3. Show that \(x = 1 - \sqrt{3}\) satisfies \(x^2 - 2x - 2 = 0\)
The term **real-number conjugates** describes expressions written \( a + b \) and \( a - b \). **Complex conjugates** are written as \( a + bi \) and \( a - bi \).

4. Write the conjugate for each.

a) \( 2 - \sqrt{11} \)  
   
   b) \( 1 + \sqrt{b} \)  
   
   c) \( \sqrt{c} - \sqrt{a} \)  
   
   d) \( 5 + \sqrt{3} \)  

5. Examine what happens when you multiply conjugates.

a) \( (x + 2)(x - 2) \)  
   
   b) \( (\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2}) \)  
   
   c) \( (6 + 2i)(6 - 2i) \)  

**Rationalizing** the numerator or denominator is the name given to the process of multiplying both the numerator and the denominator by a number that eliminates radicals from one of these positions in the fraction.

6. Rationalize the denominator in each of the following.

a) \( \frac{2}{\sqrt{3}} \)  
   
   b) \( \frac{1}{\sqrt{a}} \)  
   
   c) \( \frac{3}{4 - \sqrt{2}} \)  
   
   d) \( \frac{1}{a - \sqrt{b}} \)
To solve equations containing \( n \)th roots, take the \( n \)th power of both sides.

If \( a = b \), then \( a^n = b^n \) for any positive integer \( n \).

Even powers may introduce extraneous roots, so always check answers.

1. Use the graph to the right to solve the following equations.

   a) \( \sqrt{x+2} = 2 \)

   b) \( \sqrt{x+2} = -1 \)

2. Solve the equations from above algebraically.

   a) \( \sqrt{x+2} = 2 \)

   b) \( \sqrt{x+2} = -1 \)
3. Solve each of the following algebraically. Check your solution by substitution and by graphing. For which values of $x$ are the equations defined?

   a) $\sqrt{x - 5} = 2$

   b) $\sqrt{2x + 14} = 6$

   c) $\sqrt{3x + 1} = \sqrt{2x + 1}$